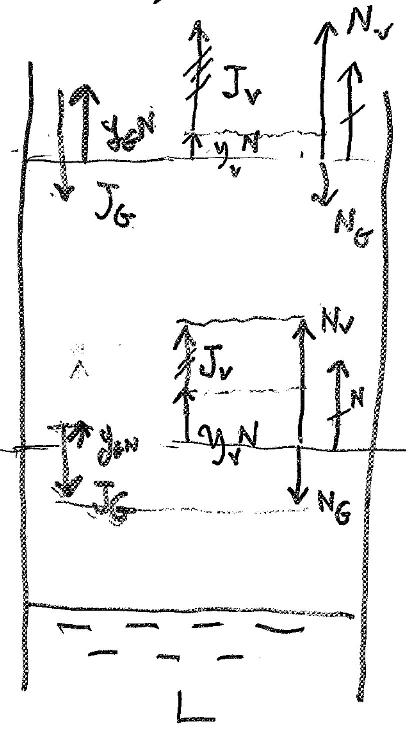
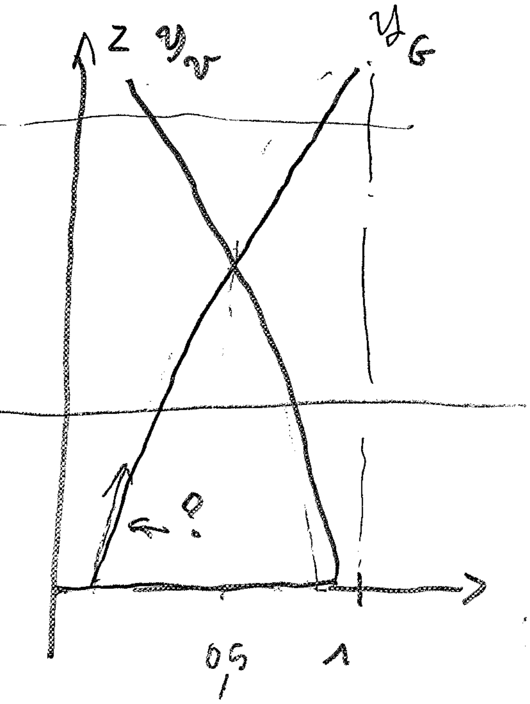
Couche et S TDM





$$pV = n\tilde{R}T \; ; \; p = \frac{n}{V}\tilde{R}T = c\tilde{R}T$$

p & T cst alors c est constant → Unités molaires.

$$c \stackrel{\Delta}{=} c_G + c_v \qquad c_v v_v + c_G v_G \stackrel{\Delta}{=} cV$$

$$N_v = c_v v_v = c_v V + \underbrace{c_v(v_v - V)}_{J_v} \qquad \nabla \cdot N_v = 0$$

$$N_G = c_G v_G = c_G V + \underbrace{c_G(v_G - V)}_{J_G} \qquad \nabla \cdot N_G = 0$$

$$J_v + J_G = 0$$

N_G = cste ; N_v = cste $\qquad N = N_v + N_G$ = cste

$$N_v = y_v N + J_v > 0$$

$$N_G = y_G N + J_G < 0$$

N_v et N_G dit par les cds d'éq. thermo.

mélanges - vapeur / liquide, Raoult $p_v = p_{sat\,v}(T_v)\, x_v \qquad x_v \approx 1$
binaires : ou gaz / liquide (dissout), Henry $p_G = H(T)\, x_G \qquad x_G \ll 1$

$$y_{v0} = \frac{p_{sat_v}(T)}{p} = \frac{p_v}{p}$$

je me donne la compo du liquide

ou

$$y_{G0} = \frac{p_G}{p} = \frac{H(T)\, x_G}{p} = \frac{p_G}{p}$$

ΔRSL

$y_{A0} = H y_A$

$$J_v = N_v - y N = - C D_{vG} \frac{dy_v}{dz}$$

$$\int_{y_{v0}}^{y_{vL}} \frac{dy_v}{y_v N - N_v} = \frac{1}{C D_{vG}} \int_0^L dz$$

$$\frac{1}{N} \ln\left(\frac{y_{vL} N - N_v}{y_{v0} N - N_v}\right) = \frac{L}{C D_{vG}}$$

$$N = \frac{C D_{vG}}{L} \ln \frac{y_{vL} - N_v/N}{y_{v0} - N_v/N}$$

$\downarrow + y_{v0} - y_{v0}$

$$\boxed{N = \frac{C D_{vG}}{L} \ln\left(1 + \frac{y_{vL} - y_{v0}}{y_{v0} - N_v/N}\right)}$$

N_v/N may be $\neq 1$

$T = 20°C$ $p = 1 atm.$

exemple

Perry Table 2-122 p.2-125 $H = 6{,}64\,10^{-4}$ atm.

— 2-5 p 2,49 $p_{sat}(20) = 17{,}535$ mm Hg

1 atm ≡ 760 mm Hg.

$$\frac{p_{sat\,v}}{p} = \frac{17{,}535}{760} = 2{,}307\,10^{-2} \quad (\sim 23\,mbar)$$

$$\frac{H(T)}{p} = 6{,}64\,10^{-4}$$

$$\begin{cases} y_v = 2{,}307\,10^{-2}\,x_v \\ y_G = 6{,}64\,10^{-4}\,x_G \\ y_v + y_G = 1 \\ x_G + x_v = 1 \end{cases}$$

à 80°C $H = 10{,}7\,10^{-4}$

$p_{sat}(80°C) = 355{,}1$ mmHg

$$y_v = \overset{a}{2{,}307\,10^{-2}}\,x_v$$

$$(1-y_v) = \overset{b}{6{,}64\,10^{-4}}\,(1-x_v)$$

$y_v = a x_v$ $(1-y_v) = b(1-x_v)$ $1-b = y_v - b x_v$

$$\begin{cases} y_v - b x_v = 1-b \\ y_v - a x_v = 0 \end{cases}$$

$+a x_v - b x_v = 1-b$

$x_v = \dfrac{1-b}{a-b}$ $x_G = \dfrac{a-b-1+b}{a-b} = \dfrac{a-1}{a-b}$

$y_v = \dfrac{a(1-b)}{a-b}$ $y_G =$

Dans les situations où ρD est constant.

Analyse en masse. (ex beaucoup d'inc)

$$n = \frac{\rho D_{vG}}{L} \ln\left(1 + \frac{\omega_{vL} - \omega_{v0}}{\omega_{v0} - n_v/n}\right)$$

ds la pratique $\rho(T,\omega)$ etc aussi → compromis.

$$n = \frac{\rho D_{vG}}{L} \ln(1 + B_v)$$

Cas d'une seule espèce transférée

$$B_v \triangleq \frac{\omega_L - \omega_{v0}}{\omega_{v0} - n_v/n} = \frac{\omega_{vL} - \omega_{v0}}{\omega_{v0} - 1} = \frac{\omega_{\infty} - \omega_i}{\omega_i}$$

inc

B_v = moteur du transfert de masse

$$j_v = n_v - \omega_v n = - \rho D_{vG} \frac{d\omega_v}{dz}.$$

$$\int_{\omega_{v0}}^{\omega_v(z)} \frac{d\omega_v}{\omega_v n - n_v} = \frac{1}{\rho D_{vG}} \int_0^z dz$$

valeur moyenne sur le film

$$\frac{1}{n} \ln\left(\frac{\omega_v n - n_v}{\omega_{v0} n - n_v}\right) = \frac{z}{\rho D_{vG}}$$

$$\frac{1}{L} \ln\left(\frac{\omega_{vL} n - n_v}{\omega_{v0} n - n_v}\right) = \frac{n\,l}{\rho D_{vG}}$$

$$\ln\left(\frac{\omega_v n - n_v}{\omega_{v0} n - n_v}\right) = \frac{nz}{\rho D_{vG}} = \frac{z}{L} \ln\left(\frac{\omega_{vL} n - n_v}{\omega_{v0} - n_v}\right) \Rightarrow \boxed{\left(\frac{\omega_v - n/n_v}{\omega_{v0} - n/n_v}\right) = \left(\frac{\omega_{vL} - n/n_v}{\omega_{v0} - n/n_v}\right)^{\frac{z}{L}}}$$

def du coefficient de T d M

$$\underline{\underline{\dot{j}_{m0}}} \stackrel{\Delta}{=} g_{mv}\,(\omega_{v0} - \omega_{vL})$$

(S) (loin)

massique

masse
molaire
loc
moyen

⚠ flux dif $\neq$ flux total

<u>En général</u> Δ (surface, extérieur, loin.)

$$\boxed{\dot{j}_{i,s} \stackrel{\Delta}{=} g_{mi}\,(\omega_{is} - \omega_{ie})}$$

$\dot{m}$

flux total ; $n_{is} = \omega_{is}\, n_s + \dot{j}_{is} = \omega_{is}\, n_s + g_{mi}\,(\omega_{is} - \omega_{ie})$

$\dot{m}$

div $n = \dot{m}$

$$\frac{n_{is}}{\dot{m}} = \omega_{is} + \frac{g_{mi}\,(\omega_{is} - \omega_{ie})}{\dot{m}}$$

$$g_{mi}\,\frac{\omega_{ie}-\omega_{is}}{\dot m} = \omega_{is}-\frac{n_{is}}{\dot m}$$

$\dot m$ calculé n'importe quelle espèce

⚠ n ← flux total

$$\boxed{\dot m = g_{mi}\left(\frac{\omega_{ie}-\omega_{is}}{\omega_{is}-n_{is}/\dot m}\right)}$$

$$\omega_{it} = \frac{n_{is}}{\dot m}$$

$$\dot m = g_{mi}\,B_{mi}$$

paramètre moteur du TdM

TdM en terme de flux total.

- mélange binaire $g_{m1}=g_{m2}$; $B_{m1}=B_{m2}$

- 1 seule espèce transférée V ; G inerte.

$$B_{mV} = \frac{\omega_{VL}-\omega_{V0}}{\omega_{V0}-1} = \frac{\omega_{G0}-\omega_{GL}}{\omega_{G0}}$$

Analyse du résultat <u>Stefan</u> en masse

$$n = \frac{\rho D_{vG}}{L} \ln\left(1 + \frac{\omega_{vL} - \omega_{v0}}{\omega_{v0} - n_v/n}\right)$$

$$n = \frac{\rho D_{vG}}{L} \ln\left(1 + B_{mv}\right)$$

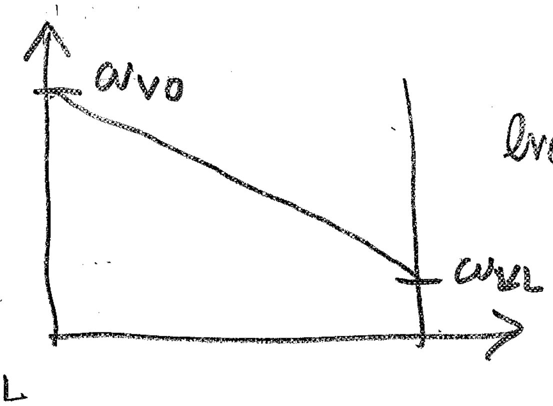
1) $|B_{mv}| \ll 1$ transfert diffusif ordinaire.
faibles TdM

si $n \approx \frac{\rho D_{vG}}{L} B_{mv} = g^*_{mv} B_{mv}$ aussi g°_{mv}

$$n_v = \cancel{\omega_v n} - \rho D_{vG} \frac{d\omega_v}{dz}$$

profil linéaire
analogue conduction

$$\frac{d}{dz} n_v \approx -\rho D_{vG} \frac{d^2 \omega_v}{dz^2} = 0$$



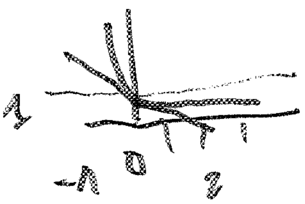
$$B_{mv} = \frac{\omega_\infty - \omega_i}{\omega_i}$$

2) B_{mv} qqc $\quad -1 < B_{mv} < 0$

$\quad 0 < B_{mv} < \infty$

$$n = \frac{\rho D_{vG}}{L} \cdot \ln(1+B_{mv}) = g_{mv} B_{mv}$$

Blowing factor



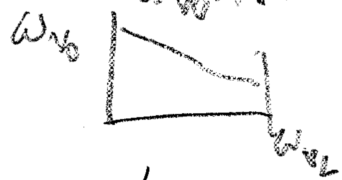
$$g_{mv} = \frac{\rho D_{vG}}{L} \frac{\ln(1+B_{mv})}{B_{mv}} = g^*_{mv} \frac{\ln(1+B_{mv})}{B_{mv}}$$

$\omega_{vL} - \omega_{v0} < 0$

$\omega_{v0} - 1 < 0$

$B_{mv} > 0$

$$\simeq g^*_{mv} \left(1 - \frac{B_{mv}}{2} + O(B_{mv}^2)\right)$$



évap.

$B_{mv} > 0 \quad n > 0$ mass away from the wall

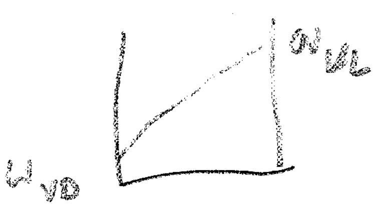
$$g_{mv} < g^*_{mv} \; ; \; 0 < \overset{BF}{\frac{\ln 1+B_m}{B_m}} < 1$$

condensation

$B_{mv} < 0 \quad n < 0$ suction ; flow towards the wall

$$1 < \frac{\ln 1+B_m}{B_m} < \infty$$

$$g_{mv} > g^*_{mv}$$



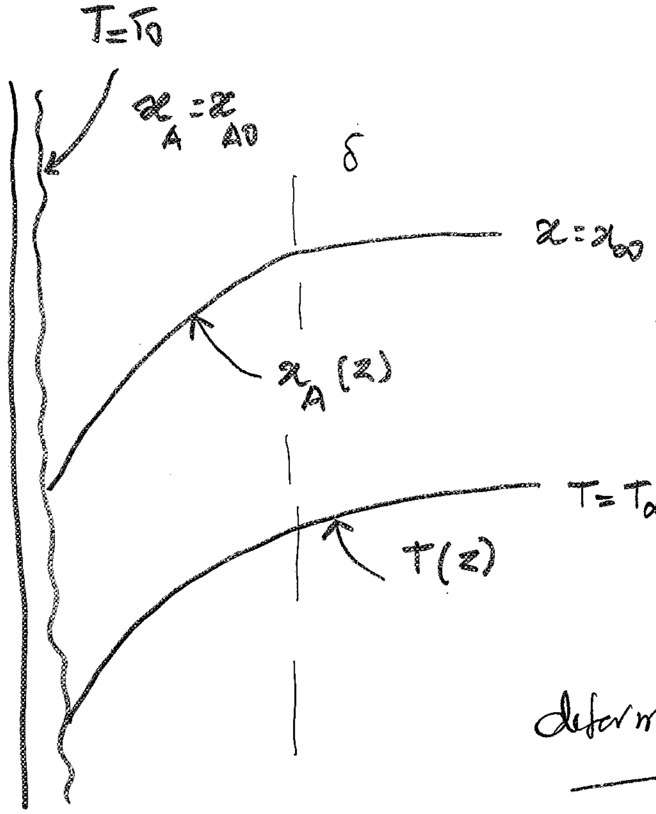
$\omega_{vL} - \omega_{v0} > 0$

$$B_{Mv} = \frac{\omega_{vL} - \omega_{v0}}{\omega_{v0} - 1}$$

= modif TdC par TdM

Théorie du film (stagnant)

Vapeur + inc



$$\begin{cases} \nabla \cdot N_A = 0 \\ \nabla \cdot q = 0 \end{cases}$$

determine x and T profiles?

$$\frac{dN_A}{dz} = 0 \qquad N_A = x_A N - c D_{AB} \frac{dx_A}{dz} \quad ; \quad N_B = 0.$$

$$N_A (1 - x_A) = - c D_{AB} \frac{dx_A}{dz} \quad ;$$

$$\int_{x_{A0}}^{x_A} \frac{dx_A}{1 - x_A} = - \frac{N_A}{c D_{AB}} \int_0^z dz$$

$$\ln \frac{1 - x_A}{1 - x_{A0}} = \frac{N_A}{c D_{AB}} z$$

$$\frac{(1-x_A)}{1-x_{A0}} = \frac{1-x_{A0}+x_{A0}-x_A}{(1-x_{A0})} = 1+\frac{x_{A0}-x_A}{(1-x_{A0})} = \exp\frac{N_A z}{cD_{AB}}$$

$x_{A0}+x_{A0}$

$$\frac{x_A-x_{A0}}{1-x_{A0}} = 1-\exp\frac{N_A z}{cD_{AB}}$$

profile NL

except $N_A \to 0$

$N_A = \frac{cD_{AB}}{\delta}\ln\left(\frac{1-x_{A\delta}}{1-x_{A0}}\right)$

$$\boxed{\frac{x_A-x_{A0}}{x_{A\delta}-x_{A0}} = \frac{1-\exp(N_A z/cD_{AB})}{1-\exp(N_A\delta/cD_{AB})}}$$

temp. $\nabla\cdot e = 0$ $\quad e = \overbrace{-k\nabla T + \sum_\alpha J_\alpha \overline{H}_\alpha}^{q} + \rho v\hat{H}$

GP $\overline{H}_\alpha = \tilde{H}_\alpha = \tilde{C}_p(T-T_0)$

averaged.

$$\rho\hat{H}v = c\tilde{H}v = v\sum_\alpha c_\alpha \overline{H}_\alpha$$

$$e = -k\nabla T + \sum_\alpha N_\alpha \overline{H}_\alpha \quad \to // z$$

$$e_0 = -k\frac{dT}{dz} + (N_A\tilde{C}_{pA} + \cancel{N_B}\tilde{C}_{pB})(T-T_0)$$

stagnant

$$e_0 = -\frac{kT}{dz} + N_A\tilde{C}_{pA}(T-T_0) \;;\; \frac{d}{dz}e_0 = 0$$

$-k r^2 + r N_A C_{pA} = 0 \qquad r\left(r - \frac{N_A C_{pA}}{k}\right) = 0$

$$-k \frac{d^2\theta}{dz^2} + N_A C_{pA} \frac{d\theta}{dz} = 0 \qquad \theta = (T - T_0)$$

$\theta = 0 \quad z = 0$

$\theta = (T_\infty - T_0) \quad z = \delta$

$$\theta = A + B \exp \frac{N_A \tilde{C}_{pA}}{k} z$$

$$\begin{cases} A + B = 0 \\ A + B \exp \frac{N_A C_{pA} \delta}{k} = (T_\infty - T_0) \end{cases} \qquad B = -A \qquad A\left(1 - \exp \frac{N_A C_{pA} \delta}{k}\right) = (T_\infty - T_0)$$

$$\boxed{T - T_0 = (T_\infty - T_0) \frac{1 - \exp(N_A C_{pA} z/k)}{1 - \exp(N_A C_{pA} \delta/k)}}$$

profile NL

some $N_A \to 0$

at the wall

$$-k \frac{dT}{dz}\bigg|_{z=0} = \frac{(T_\infty - T_0) \; N_A C_{pA}}{1 - \exp(N_A C_{pA} \delta/k)}$$

$\exp \varepsilon \sim 1 + \varepsilon + \frac{\varepsilon^2}{2}$

$1 - \exp \varepsilon \sim -\varepsilon - \frac{\varepsilon^2}{2}$

$N_A \to 0 \qquad -\frac{(T_\infty - T_0)\,\delta}{k}$

$$\frac{-\varepsilon}{-\varepsilon - \varepsilon^2/2 + O(\varepsilon^3)}$$

$$\frac{1}{1 + \varepsilon/2 + O(\varepsilon^2)}$$

$$\boxed{\frac{-k(dT/dz)_{z=0}}{-k(dT/dz)^*_{z=0}} = \frac{-(N_A C_{pA} \delta/k)}{1 - \exp N_A C_{pA} \delta/k}} > 1$$

cond $N_A < 0$

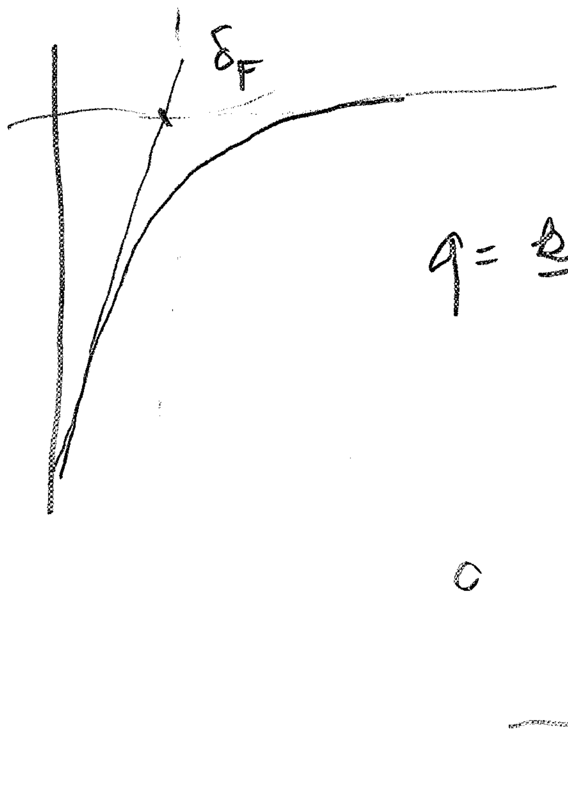
$1 - \frac{\varepsilon}{2} + O(\varepsilon^2)$

Condens. HT — increase H flux

Analogie complète

$$\frac{1}{2} f = St \, Pr^{2/3} = St_m \, Sc^{2/3}$$

$$Nu = \frac{hL}{k} = \frac{qL}{k \Delta T}$$



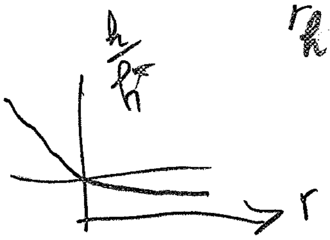
$$q = \frac{k \Delta T}{\delta_f} \qquad Nu = \frac{L}{\delta_f}$$

o

$\rightarrow r_h$

19.4-16 → def coef T
19.4.17

$$\boxed{\frac{h}{h^*} = \frac{-r_h}{1-\exp(r_h)}}$$



$$r_h = \frac{N_A c_{pA} \delta}{k} = \frac{N_A c_{pA} (T_\infty - T_0)}{\frac{k (T_\infty - T_0)}{\delta}} = \frac{\frac{N_A}{\dot{m}} c_{pv} (T_\infty - T_0)}{\phi^*} = \frac{\frac{N_A}{\dot{m}} c_{pv}}{h^*}$$

cond $\dot{N}_A < 0$

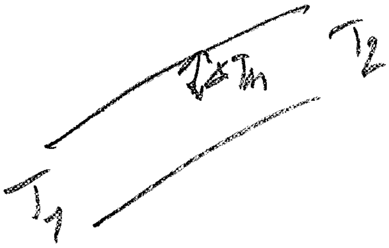
$$\boxed{\frac{g_{mv}}{g_{mv}^*} = \frac{\ln(1 + B_{vm})}{B_{vm}}}$$

$$B_{vm} = \frac{\omega_{v\infty} - \omega_{v0}}{\omega_{v0} - 1} = \frac{\omega_\infty - \omega_0}{\omega_0}$$

incond.

Résultat analogue pour le frottement

$$\frac{\tau_w}{\tau_w^*} = \frac{-r_m}{1-\exp(r_m)} \quad ; \quad r_m = \frac{\dot{m}\, u_\infty}{\tau_w^*}$$



Vermeer & Dalhe.

$\dot{M} c_p \Delta T = Q = qS$

$A G c_p \Delta T = qS \qquad \Delta T_m -$

$$\left(\frac{\Delta T}{\Delta T_m}\,\frac{A}{S}\right) = \frac{q}{G c_p \Delta T_m} = \frac{h}{G c_p} = St$$

Colburn

def du Nbe de Sherwood. Nusselt

flux dif. flux dif.

j_A échelle diffusion | q

surface ext.

$$\frac{\rho D_{AB}(\omega_{AS} - \omega_{AE})}{L} \qquad \Big| \qquad \frac{k(T_S - T_E)}{L}$$

def du coef. de T $\quad j_A = g_{mA}(\omega_{AS} - \omega_{AE}) \qquad q = h\,(T_S - T_E)$

en termes coef TAM

$$Sh \triangleq \frac{g_{mA} L}{\rho D_{AB}} \qquad Nu = \frac{hL}{k}$$

$$= \frac{j_A L}{\rho D_{AB}(\omega_{AS} - \omega_{AE})} \quad \text{flux dif.}$$

$$n_A = g_{mA} \frac{\omega_{AS} - \omega_{AE}}{1 - \omega_{AS}}$$

$$= \frac{n_A L (1 - \omega_{AS})}{\rho D_{AB}(\omega_{AS} - \omega_{AE})} \quad \text{flux total.}$$

P223

– o –

Stagnant film model

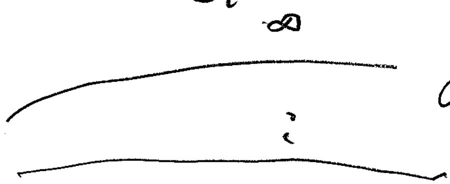
Voir aussi NACA TN 3208 (1954) Mickley Ross Squyers Stewart.

– correction for high transfer rates BSL p. 708

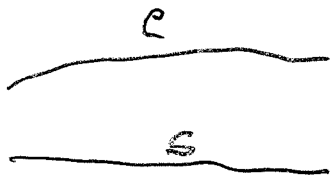
– Vernier Delbe 1984 p. 41 $\theta_d = \frac{\ln(1+R)}{R}$

– Lienhardt p. 658 $\theta_d = \ln \frac{1+B_{mi}}{B_{mi}}$

$$R = \frac{\dot{m}(\omega_i - \omega_\infty)}{J_i} = \frac{\omega_\infty - \omega_i}{\omega_i} \; ; \; B_{mi} = \frac{\omega_{i,e} - \omega_{i,s}}{\omega_{i,s} - 1}$$



$\omega = \text{inc}$



– theory du film BSL ex 19.4-1 p. 592

– conv mat BSL p. 346