Analogies I

Start.

Masse α $\quad \nabla\cdot \rho\omega_\alpha v = -\nabla\cdot j_\alpha + r_\alpha.$

Chaleur $\quad \nabla\cdot \rho h v = -\nabla\cdot q + q'''$

qdm $\quad \nabla\cdot \rho v v = -\nabla\cdot \Pi = -\nabla p + \nabla\cdot \mathbb{V}$

$$j_\alpha = -\rho D \nabla \omega_\alpha \qquad r_\alpha = 0$$

$$q = -k\nabla T + \cancel{\Sigma H_\alpha j_\alpha} \qquad q''' = 0 \quad \text{CL}$$

$$\mathbb{V} = \mu(\nabla v + v\nabla) \qquad \nabla p = 0$$

$\left.\begin{array}{l} U \quad \Delta T \\ \omega = SD \end{array}\right\}$ Dirichlet BC

$$\nabla\cdot \omega_A v' = -\frac{D}{UL}\nabla^2\omega_A = -\frac{1}{Pe'_m}\nabla^2\omega_A$$

$$\nabla\cdot \theta v' = -\frac{k}{\rho C_p U L}\nabla^2\theta = -\frac{1}{Pe}\nabla^2\theta$$

$$\nabla\cdot v'v' = -\frac{\mu}{\rho U L}\nabla^2 v' = -\frac{1}{Re}\nabla^2 v'$$

$$Re = \frac{\rho U L}{\mu} = \frac{UL}{\nu} \qquad Pe' = \frac{\rho C_p U L}{k} = \frac{UL}{\alpha} \qquad Pe'_m = \frac{UL}{D}$$

$$\frac{Pe'}{Re} = \frac{\nu}{\alpha} = \frac{\mu C_p}{k} \triangleq Pr \qquad Pe' = Re\,Pr$$

$$\frac{Pe'_m}{Re} = \frac{\nu}{D} \triangleq Sc$$

1) Analogie TdM et T de C

$$Nu = \frac{hL}{k} = F(Re, Pr) \qquad \forall\, Sc \,\&\, Pr$$

$$Sh = \frac{gL}{\rho D} = F(Re, Sc) =$$

2) Analogie Td Ch et frottement

2,1) $Pr = 1$

$$\frac{1}{2} f = \frac{\tau}{\rho U^2} = F(Re)$$

Reynolds

$Pr \sim 1$

$$St = \frac{q}{\rho C_p U \Delta T} = \frac{h}{\rho C_p U} = F(Re)$$

$$Nu = \frac{qL}{k\Delta T} = \frac{qL}{\rho C_p U \Delta T} \frac{\rho C_p U \cancel{\Delta T}}{k \cancel{\Delta T}}$$

$$= St\, Pé$$

2,2) $0{,}5 < Pr < 50 - \infty)$

$$St = F(Re)\, G(Pr)$$

Colburn from expe have shown

$$j_h = St\, Pr^{2/3} = \frac{1}{2} C_f .$$

⚠ quasi 1D flow

$$St_m = \frac{j}{\rho U \Delta\omega} = \frac{g}{\rho U}$$

$$j_m = St_m \, Sc^{2/3}$$

$$\boxed{\frac{1}{2} f = \underbrace{St \, Pr^{2/3}}_{j_h} = \underbrace{St_m \, Sc^{2/3}}_{j_m}}$$

Chilton – Colburn

Origine du $Pr^{2/3}$ = Analyse profils de T et U ds CL laminaire

$$\frac{u}{U} = f\left(\frac{y}{\delta}\right) \qquad \delta = \text{ép CL cin.}$$

$$\theta = f\left(\frac{y}{\delta_T}\right) \qquad \frac{\delta_T}{\delta} \simeq Pr^{-1/3} \quad 0{,}6 < Pr < 50$$

$$\tau = \frac{\mu U}{\delta} f'(0) \; ; \; \frac{1}{2} f = \frac{\tau}{\rho U^2} = \frac{1}{Re_\delta} f'(0)$$

$$q = \frac{k \Delta T}{\delta_T} f'(0) \; ; \; Nu = \frac{q \delta_T}{k \Delta T} = f'(0)$$

$$\frac{1}{2} f \, Re_\delta = Nu_{\delta_T} = St \, Pe_{\delta_T} = St \, Re_{\delta_T} Pr$$

$$\frac{1}{2} f = St\, Pr \frac{Re_{\delta_T}}{Re_\delta} = St\, Pr \frac{\delta_T}{\delta}$$

$$\approx St\, Pr^{2/3} \quad \square$$

<u>Exemples</u> j'ai <u>f</u> $\rightarrow$ je cherche <u>Nu</u>

$$Nu = St\, Pe' \qquad \rightarrow Nu = St\, Re\, Pr$$

$$Pe' = Re\, Pr \qquad St = \frac{Nu}{Re\, Pr}$$

$$\boxed{\frac{1}{2} f = St\, Pr^{2/3} = \frac{Nu}{Re\, Pr^{1/3}} = j_h} \quad \text{Colburn factor}$$

$$Nu = \frac{1}{2} f\, Re\, Pr^{1/3}$$

ex. conduite éc T.

$f = 0{,}046\, Re^{-0,2}$ Blasius

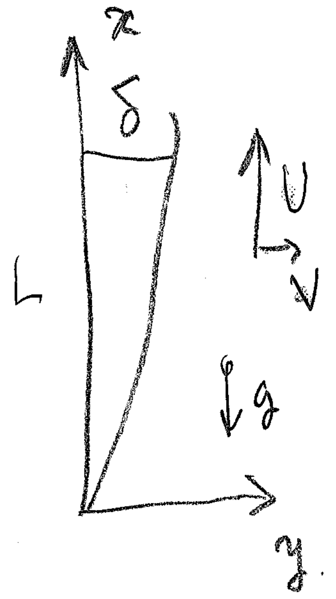
$Nu = 0{,}023\, Re^{0,8}\, Pr^{1/3}$ $\qquad Nu = G(Re, Pr)$

$Sh = 0{,}023\, Re^{0,8}\, Sc^{1/3}$ $\qquad Sh = G(Re, Sc)$

$$\begin{pmatrix} Nu_L \\ \\ Sh_L \end{pmatrix} = 0{,}037 \begin{pmatrix} Pr^{1/3} \\ \\ Sc^{1/3} \end{pmatrix} Re_L^{0{,}8}$$

$$5\,10^5 < Re_L < 10^7$$

Conv naturelle Approx CL



$\nabla . \rho v = 0 \qquad \dfrac{\partial \rho u}{\partial x} + \dfrac{\partial \rho v}{\partial y}$

$\nabla . \rho v v = \nabla . \Pi + \rho g \qquad \dfrac{\partial \rho u^2}{\partial x} + \dfrac{\partial \rho u v}{\partial y} = \mu \dfrac{\partial^2 u}{\partial y^2} - \dfrac{dp}{dx} - \rho g.$

$p = p(x) \; ; \; \dfrac{dp}{dx} = -\rho_\infty g$

$$c_p \left(\frac{\partial \rho u T}{\partial x} + \frac{\partial \rho v T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2}$$

$\nabla . \rho v \omega_A = -\nabla . j$

$$\frac{\partial \rho u \omega_A}{\partial x} + \frac{\partial \rho v \omega_A}{\partial y} = \rho D \frac{\partial \omega_A}{\partial y^2}$$

Boussinesq.

$$\rho = \rho_\infty + \left.\frac{\partial \rho}{\partial T}\right)_{\omega,\infty} (T - T_\infty) + \left.\frac{\partial \rho}{\partial \omega}\right)_{T,\infty} (\omega - \omega_\infty)$$

$$\rho_\infty - \rho = -\frac{\partial \rho}{\partial T}(T - T_\infty) - \frac{\partial \rho}{\partial \omega}(\omega - \omega_\infty)$$

$\omega = \omega_A$

$$\rho_\infty - \rho = \rho_\infty \left(\beta (T - T_\infty) - \gamma (\omega - \omega_\infty) \right)$$

$$\beta = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_{p, \omega_\infty} = \frac{1}{T_\infty} \quad \text{pour GP}$$

$$\frac{p}{\rho} = \frac{\tilde{R} T}{M}$$

$$\frac{1}{M} = \frac{\omega_A}{M_A} + \frac{\omega_B}{M_B}$$

ω $1-\omega$

$$\gamma = \frac{1}{\rho}\left(\frac{\partial \rho}{\partial \omega}\right)_{T, p, \infty} = \frac{1}{\frac{M_A}{M_A - M_V} - \omega_\infty}$$

$$\frac{\partial \rho u^2}{\partial x} + \frac{\partial \rho u v}{\partial y} = \mu \frac{\partial^2 u}{\partial y^2} + \overbrace{\rho_\infty g \left(\beta (T - T_\infty) + \gamma (\omega_\infty - \omega) \right)}^{(\rho_\infty - \rho) g \quad B}$$

$$B = \rho g \beta \Delta T$$

$$\rho \left(\frac{U}{L} \quad \frac{V}{\delta} \right) = 0$$

$$\rho U \left(\frac{U}{L} \quad \frac{V}{\delta} \right) = \frac{\mu U}{\delta^2} \quad B \quad \cancel{\frac{\mu U}{L^2}}$$

$$\theta\left(\frac{\delta^2}{L^2}\right)$$

$$\rho C_p \Delta T \left(\frac{U}{L} \quad \frac{V}{\delta} \right) = \frac{k \Delta T}{\delta^2}$$

3 eqs δ, U, V ?

Energie + masse $\quad \frac{U}{L} = \frac{V}{\delta} = \frac{\alpha}{\delta^2}$ ①

1) $U = \frac{\alpha L}{\delta^2}$ 2) $U = \frac{B\delta^2}{\mu}$

$$\frac{\alpha L}{\delta^2} = \frac{B\delta^2}{\mu} \Rightarrow \delta^4 = \frac{\mu \alpha L}{B}$$

qdm $\quad \frac{\mu U}{\delta^2} = B$ ②

$$U^2 = \frac{B^2\delta^4}{\mu^2} = \frac{B^2}{\mu^2}\frac{\mu\alpha L}{B} = \frac{B\alpha L}{\mu}$$

$$V^4 = U^4\frac{\delta^4}{L^4} = \frac{B^2\alpha^2 L^2}{\mu^2}\frac{\mu\alpha L}{B}\frac{1}{L^4} = \frac{B\alpha^3}{L\mu}$$

$$\delta = \left(\frac{\alpha\mu L}{B}\right)^{1/4} \qquad U = \left(\frac{B\alpha L}{\mu}\right)^{1/2}$$

$$\theta = \frac{T - T_\infty}{T_w - T_\infty}$$

$$V = \left(\frac{B\alpha^3}{\mu L}\right)^{1/4}$$

$$u' = \frac{u}{U} \qquad v' = \frac{v}{V}$$

$$\xi = \frac{x}{L} \qquad \eta = \frac{y}{\delta}$$

CL $\begin{cases} \eta = 0 & u' = v' = 0 \quad \theta = 1 \\ \eta \to \infty & u' = 0 \quad \theta = 0 \end{cases}$

$$\frac{\partial u'}{\partial \xi} + \frac{\partial v'}{\partial \eta}$$

$$\frac{1}{Pr}\left(u'\frac{\partial u'}{\partial \xi} + v'\frac{\partial u'}{\partial \eta} \right) = \frac{\partial^2 u'}{\partial \eta^2} + \theta$$

$$\left(u'\frac{\partial \theta}{\partial \xi} + v'\frac{\partial \theta}{\partial \eta} \right) = \frac{\partial^2 \theta}{\partial \eta^2}$$

$$u', v', \theta = f(Pr), \xi, \eta).$$

$$q_L = \frac{1}{L}\int_0^L -k\frac{\partial T}{\partial y}\,dx$$

$$= \frac{k\Delta T}{L}\,\frac{1}{\delta}\int_0^1 \left.\frac{\partial \theta}{\partial \eta}\right)_{\eta=0} d\xi$$

$$q_L = k(T_w - T_\infty)\left(\frac{B}{\alpha\mu L}\right)^{1/4} C(P_r)$$

$L \nearrow$ cste

$1 < P_r < \infty$

$$Nu_L = \frac{q_L L}{k(T_w - T_\infty)} = C(P_r)\left(\frac{BL^3}{\alpha\mu}\right)^{1/4}$$

$$\frac{\rho g \beta (T_w - T_\infty) L^3}{\alpha\mu} = Ra_L$$

$$\boxed{Nu_L = C(P_r)\,(Ra_L)^{1/4}}$$

Grashof.

$$Ra = Gr\,Pr \qquad Gr = \frac{\rho g \beta (T_w - T_\infty) L^3}{\mu^2}$$

	(air)					
Pr	0,73	1	10	100	1000	∞
C	0,518	0,535	0,620	0,653	0,665	0,670

En régime Laminaire,
Nu_L=0,555 Ra^(1/4)

Conv Turbulente Eckert & Jackson

$$Nu_L = 0{,}0210\,(Gr\,Pr)^{2/5}$$

Vérification qualité approx CL $\frac{\delta^2}{L^2} \ll 1$

$\frac{\delta^4}{L^4} = \frac{\alpha\mu}{BL^3}$; $\frac{\delta^2}{L^2} = \left(\frac{\alpha\mu}{BL^3}\right)^{1/2} = Ra^{-1/2}$ $Ra \to \infty$; OK

– Ra et Gr = ~~$\delta g H$ visqueux~~

$\tau = \mu \frac{du}{dy}$

293 —— 300 $\boxed{\rho = 1{,}177 \quad Cp : 1007 \quad \mu\ 1{,}857\ 10^{-5} \quad \nu = 1{,}518\ 10^{-5} \quad k = 0{,}02623}$

313

$\alpha = 2{,}213\ 10^{-5}\ m^2/s$

$Pr = 0{,}713$

CL $\frac{\delta}{L} = Re^{-1/2}$ $\frac{\delta^2}{L^2} = \frac{1}{Re}$ $\frac{\rho U^2}{L} = \frac{\mu U}{\delta^2}$

$\delta^2 = \frac{\mu U L}{\rho U^2} = L^2 \frac{\mu}{\rho U L}$

$Ra \neq \frac{\text{moteur}}{\text{visc}} = 1$

T de ch et de Masse simultanés

$$G_r^* = \frac{\rho g\left[\beta(T_W - T_\infty) + \gamma(\omega_\infty - \omega_W)\right] L^3}{\mu^2}$$

Termes négligés

$$\rho c_p \frac{DT}{Dt} \qquad k\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 - \frac{T}{\rho}\left.\frac{\partial \rho}{\partial T}\right)_{p,\omega} \frac{Dp}{Dt}$$

$$\beta T \frac{Dp}{Dt} = \beta T v . \nabla p$$

$$\frac{Dp}{Dt} = v . \nabla p \quad = \quad - \rho_\infty \beta T u g .$$

$$\rho c_p \Delta T\left(\frac{U}{L} \quad \frac{V}{\delta}\right) = \frac{k\,\Delta T}{\delta^2} \quad \frac{\mu U^2}{\delta^2} \quad \rho\beta T_0 U g$$

1 $\qquad \dfrac{\mu U^2}{\delta^2}\dfrac{1}{\rho c_p \Delta T}\dfrac{L}{U} \qquad \dfrac{\rho\beta T_0 U g}{\rho c_p \Delta T} \cdot \dfrac{L}{U}$

I) $\qquad$ II)

$$\frac{\mu U L}{\delta^2 \rho C_p \Delta T} = \frac{\mu \left(\frac{B\alpha L}{\mu}\right)^{1/2} L}{\left(\frac{\alpha \mu L}{B}\right)^{1/2} \rho\, C_p \Delta T} =$$

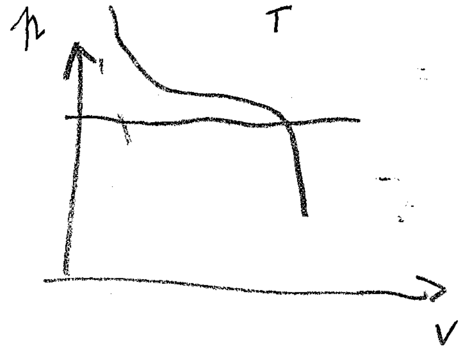
I)

$$= \frac{B \cancel{\mu}^{0} \cancel{\alpha}^{0} L}{\rho C_p \Delta T} = \frac{\cancel{\rho} g \beta \cancel{\Delta T} L}{\cancel{\rho} C_p \cancel{\Delta T}} \qquad \frac{g\beta L}{C_p} \ll 1$$

II)

$$\frac{\cancel{\rho}\, \beta T_0 \cancel{U} g L}{\cancel{\rho} C_p \Delta T \cancel{U}} \qquad \frac{\beta T_0 g L}{C_p \Delta T} = \Phi \ll 1$$

gd élancement
ΔT petit + pt critique



He 20bar 2 T 10

$\underline{\text{air}}$ $\underline{1\,m}$ $\frac{g\beta L}{C_p}$ $\frac{10 \cdot 1/300}{1000} = 10^{-2}\, 0{,}3\, 10^{-3} = 3\, 10^{-6}$ I)

$\Delta T = 10$ $\frac{\beta T_0 g L}{C_p \Delta T}$ $\frac{10}{1000\ 10} = 10^{-3}$ II)

He 20b 8K $\left.\frac{\partial \rho}{\partial T}\right)_p = 10$ $\beta = \frac{1}{\rho}\frac{\partial \rho}{\partial T} = \frac{10}{150} = 7\,10^{-2}$ $\frac{10\ 7\,10^{-2}\ 1}{5000}$ $1{,}4\,10^{-4}$ I)

$\Delta T = 0{,}1K$ $C_p = 5000$ $\frac{7\,10^{-2} \cdot 8 \cdot 10}{5000\ 0{,}1} = 0{,}42$

Pons M, Le Quéré P,

P Lequéré M Pons 2005

127-132 (I)

CR Mec 333 133-138 (II)

(1951)

Eckert & Jackson NACA TR 1015

Bejan A, 1984 Convection Heat transfer

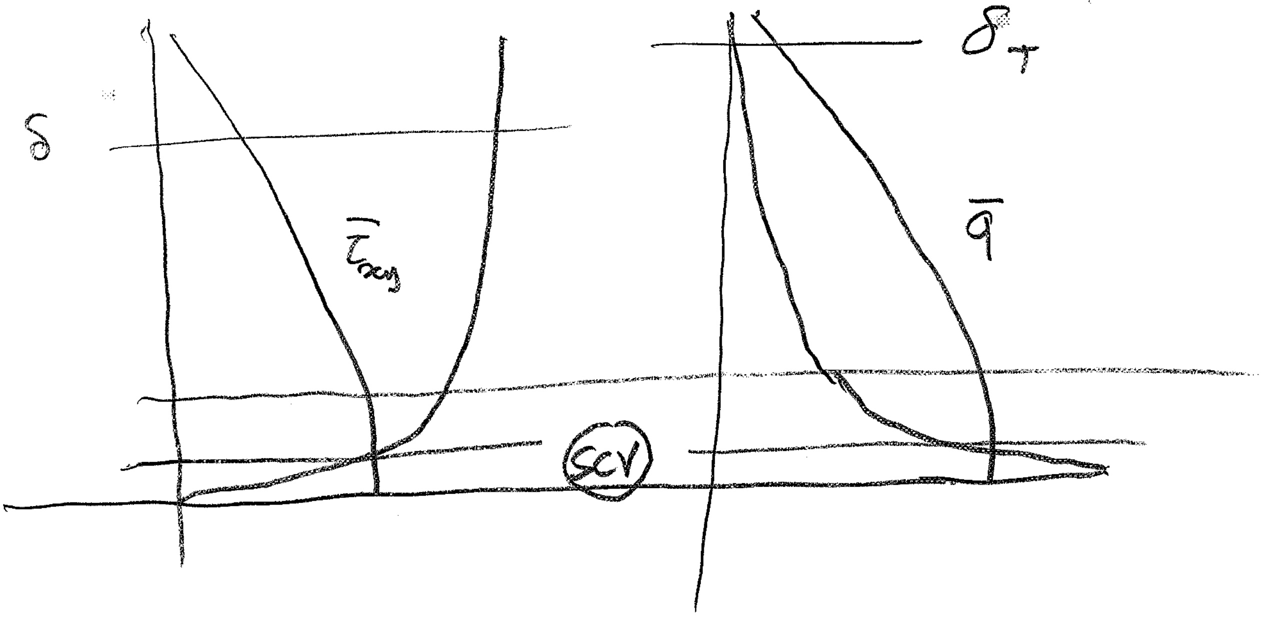
John Wiley & Sons.

Bejan A, 1993 Heat transfer

John Wiley & Sons.

2S9 260 606 — 14 —

Analogie frottement — TdC



SCV

$$0 = \frac{d}{dy}\left[-(k + \rho C_p \varepsilon_H)\frac{d\overline{T}}{dy}\right] = 0 \qquad -(k + \rho C_p \varepsilon_H)\frac{d\overline{T}}{dy} = q_w$$

$$0 = \frac{d}{dy}\left[(\mu + \rho \varepsilon_M)\frac{d\overline{u}}{dy}\right] = 0 \qquad (\mu + \rho \varepsilon_M)\frac{d\overline{u}}{dy} = \tau_w$$

$$\frac{\rho(\nu + \varepsilon_M)}{\rho C_p(\alpha + \varepsilon_H)} \frac{d\overline{u}}{d\overline{T}} = -\frac{\tau_w}{q_w} \qquad P_r = 1 \quad P_{r_t} = 1$$

$$\frac{1}{C_p}\frac{d\overline{u}}{d\overline{T}} = -\frac{\tau_w}{q_w} \Big| f(P_r)$$

$$\frac{u_\infty}{C_p(T_\infty - T_w)} = \frac{-\tau_w}{q_w} \Rightarrow q_w \propto \tau_w$$

Stanton

Analogie TdC et frot.

$$St = \frac{q_w}{\rho C_p (T_w - T_\infty) U_\infty} = \frac{h}{\rho C_p U_\infty} = \frac{\Delta T}{\Delta T_m} \frac{S}{A}$$

$$C_f = \frac{\tau_w}{\frac{1}{2} \rho U_\infty^2}$$

$$\frac{\cancel{u_\infty}}{\cancel{C_p (T_\infty - T_w)}} = \frac{C_f \frac{1}{2} \rho \cancel{u_\infty^2}}{St \, \cancel{\rho C_p (T_\infty - T_w) U_\infty}} \qquad \frac{\frac{1}{2} C_f}{St} = 1 \quad \Big| = f(P_r)$$

Analogie de Reynolds $P_r = 1$

$$\boxed{\frac{1}{2} C_f = St \, P_r^{2/3}} \quad \begin{cases} \text{expé} \\ \text{CL} \end{cases}$$

Analogie de Colburn ⚠ CL pas de traînée de forme

$5 < P_r < \infty$

$$St = \frac{q_w L}{k (T_w - T_\infty)} \frac{k \cancel{T_w - T_\infty}}{L} \frac{1}{\rho C_p \cancel{(T_w - T_\infty)} U_\infty} = Nu \left(\frac{\alpha}{U_\infty L}\right)^{1/Pe}$$

$$= Nu \left(\frac{\nu}{U_\infty L}\right)^{1/Re} \left(\frac{\alpha}{\nu}\right)^{1/P_r}$$

$$St = \frac{Nu}{Pe} = \frac{Nu}{Re.Pr} =$$

Analogie TdM et TdC

$$\begin{cases} Nu & - & Sh \\ Pr & - & Sc \end{cases}$$

Analogie de Reynolds $Sc = 1 \quad Pr = 1$

$$\frac{1}{2} C_f = \underbrace{\frac{Nu}{Re\, Pr}}_{St} = \underbrace{\frac{Sh}{Re\, Sc}}_{St_m}$$

Analogie de Chilton-Colburn conv. forcée

$$\frac{1}{2} C_f = \underbrace{\frac{Nu}{Re\, Pr^{1/3}}}_{j_H} = \underbrace{\frac{Sh}{Re\, Sc^{1/3}}}_{j_D}$$

$$St\, Pr^{2/3} \qquad St_m\, Sc^{2/3}$$

ex : en conduite $\qquad C_f = 0{,}046\, Re^{-0,2}$ (Re ?)

$$0{,}023\, Re^{-0,2} = \frac{Nu}{Re\, Pr^{1/3}} \qquad Nu = 0{,}023\, Re^{0,8}\, Pr^{1/3}$$

D-B. eq.

CL T $\qquad C_{f_x} = 0{,}0592\, Re_x^{-0,2}$