

- Techniques de mesure
(taux de vide)
- Modèles de taux de vide.
Calcul pratique.

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RAPPELS des OPERATEURS de MOYENNE.

- Fonction indicatrice de phase

$$X_k(r, t) = 1 \text{ si } M(r) \in \text{phase } k$$

- Moyennes spatiales

phasique : $\langle f \rangle_n = \frac{1}{\mathcal{D}_{kn}} \int_{\mathcal{D}_{kn}} f d\mathcal{D}_{kn}$

globale : $\langle f \rangle_n = \frac{1}{\mathcal{D}} \int_{\mathcal{D}} f d\mathcal{D} \quad (\mathcal{D} = \bigcup_k \mathcal{D}_k)$

- Moyennes temporelles

conditionnelle : $\overline{f}^x = \frac{1}{T_k} \int_{[T_k]} f dt$

globale : $\overline{f} = \frac{1}{T} \int_{[T]} f dt$

$[T]_k$ ensemble des instants où $X_k = 1 \cap [T]$

$[T]$ intervalle d'intégration $[t - \frac{T}{2}, t + \frac{T}{2}]$

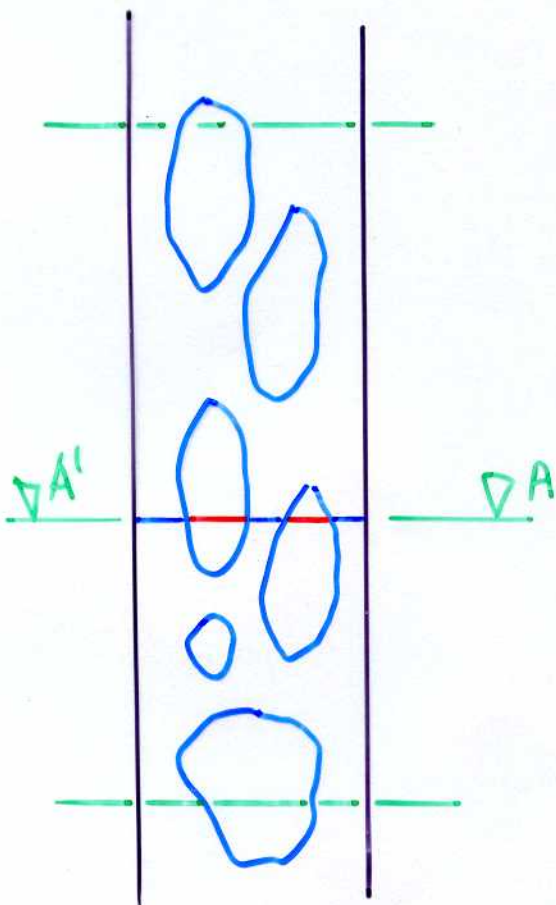
IDENTITÉ : $R_{kn} \langle f \rangle_n = \alpha_k \overline{f}^x$

LES DEFINITIONS du TAUX de VIDE (α)

- Taux de vide local α_G

temps de présence moyen : $\alpha_G \triangleq \frac{T_G}{T_G + T_L} = \frac{T_G}{T}$

- Fractions Spatiales de gaz/vapeur/vide instantanées



- fraction linéique instantanée

$$R_{G1} \triangleq \frac{L_G}{L_G + L_L} = \frac{L_G}{L}$$

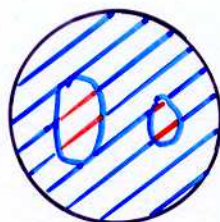
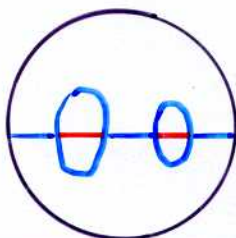
- fraction surfacique instant.

$$R_{G2} \triangleq \frac{A_G}{A_G + A_L} = \frac{A_G}{A}$$

- fraction volumique instant.

$$R_{G3} \triangleq \frac{V_G}{V_G + V_L} = \frac{V_G}{V}$$

Coupe
A-A'



RELATIONS FONDAMENTALES IDENTITÉS

- Rappel Commutativité des opérateurs

$$\overline{R_{kn} \langle f \rangle_n} = \langle \alpha_k^x f^x \rangle_n \quad (\text{avec } f \equiv 1)$$

- Taux de présence (vide) moyen

+ sur une ligne

$$\overline{R_{G1}} \triangleq \frac{1}{T} \int_{[T]} R_{G1} dt \equiv \frac{1}{L} \int_L \alpha_G dL$$

+ sur une surface

$$\overline{R_{G2}} \triangleq \frac{1}{T} \int_{[T]} R_{G2} dt \equiv \frac{1}{A} \int_A \alpha_G dA$$

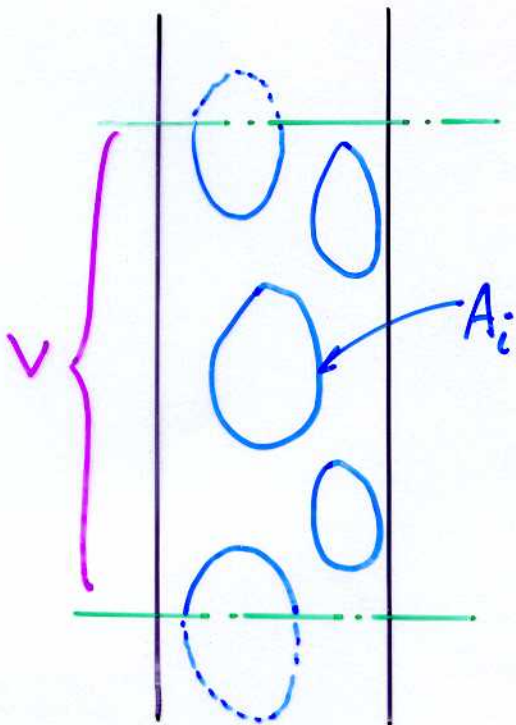
+ dans un volume

$$\overline{R_{G3}} \triangleq \frac{1}{T} \int_{[T]} R_{G3} dt \equiv \frac{1}{V} \int_V \alpha_G dV$$

- Les définitions sont précises.

AUTRES DÉFINITIONS

- Aire interfaciale volumique (instantanée)



$$\overline{\Gamma}_3 \triangleq \frac{A_i}{V}$$

- Aire interfaciale locale

$$\gamma = \sum_{\substack{\text{discont.} \\ \in [T]}} \frac{1}{|\mathbf{v}_i \cdot \mathbf{n}_k|}$$

- Identité :

$$\overline{\Gamma}_3 \equiv \langle \gamma \rangle_3$$

TECHNIQUES de MESURE du TAUX de VIDE.

- Taux de vide local

Sondes électriques } α et γ
Sondes optiques }

- Taux de présence sur une ligne

Atténuation des Rayons X ou γ

- Taux de présence surfacique

Rayons X ou γ : one-shot technique

Densitomètre multi faisceaux

Diffusion de neutrons

Densitomètre à impédance (ep. de film)

- Taux de vide moyen (volumique)

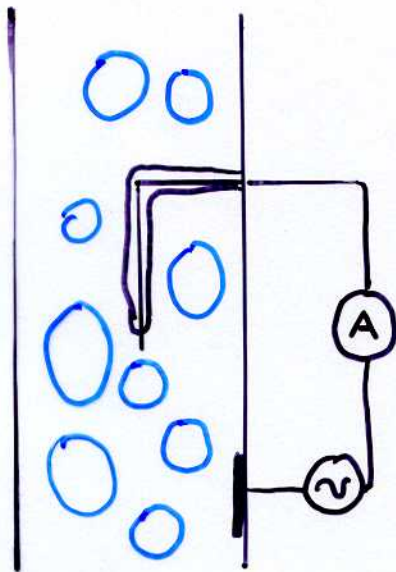
Vannes à fermeture rapide

Méthodes ultrasonores (α , ρ_i , d_{sm})

TAUX de VIDE LOCAL

Mesure de la FIP ($X_R(r, t)$)

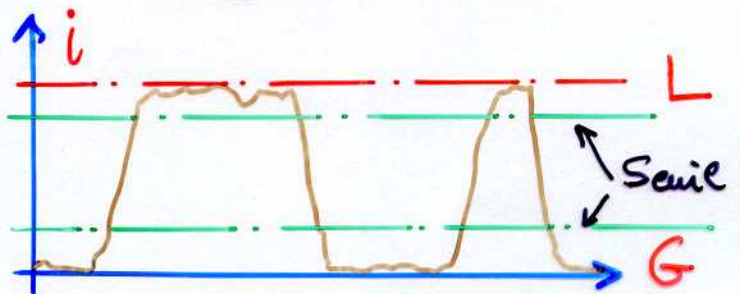
- Sondes résistives (résistivité)



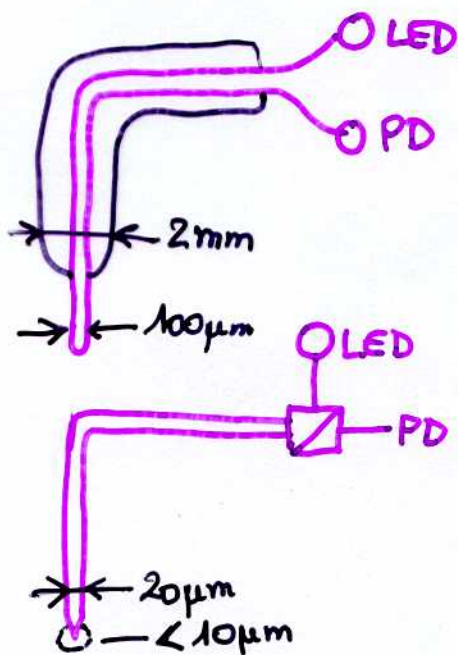
Milieu continu conducteur

Phase dispersée volante

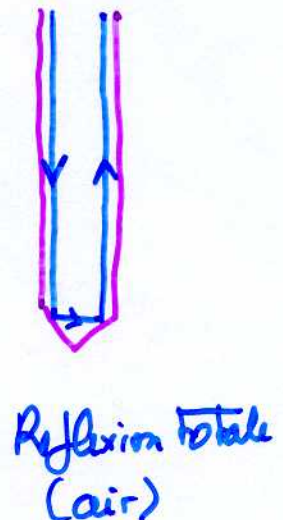
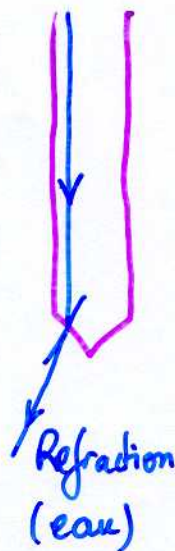
Seuillage : α (seuil)



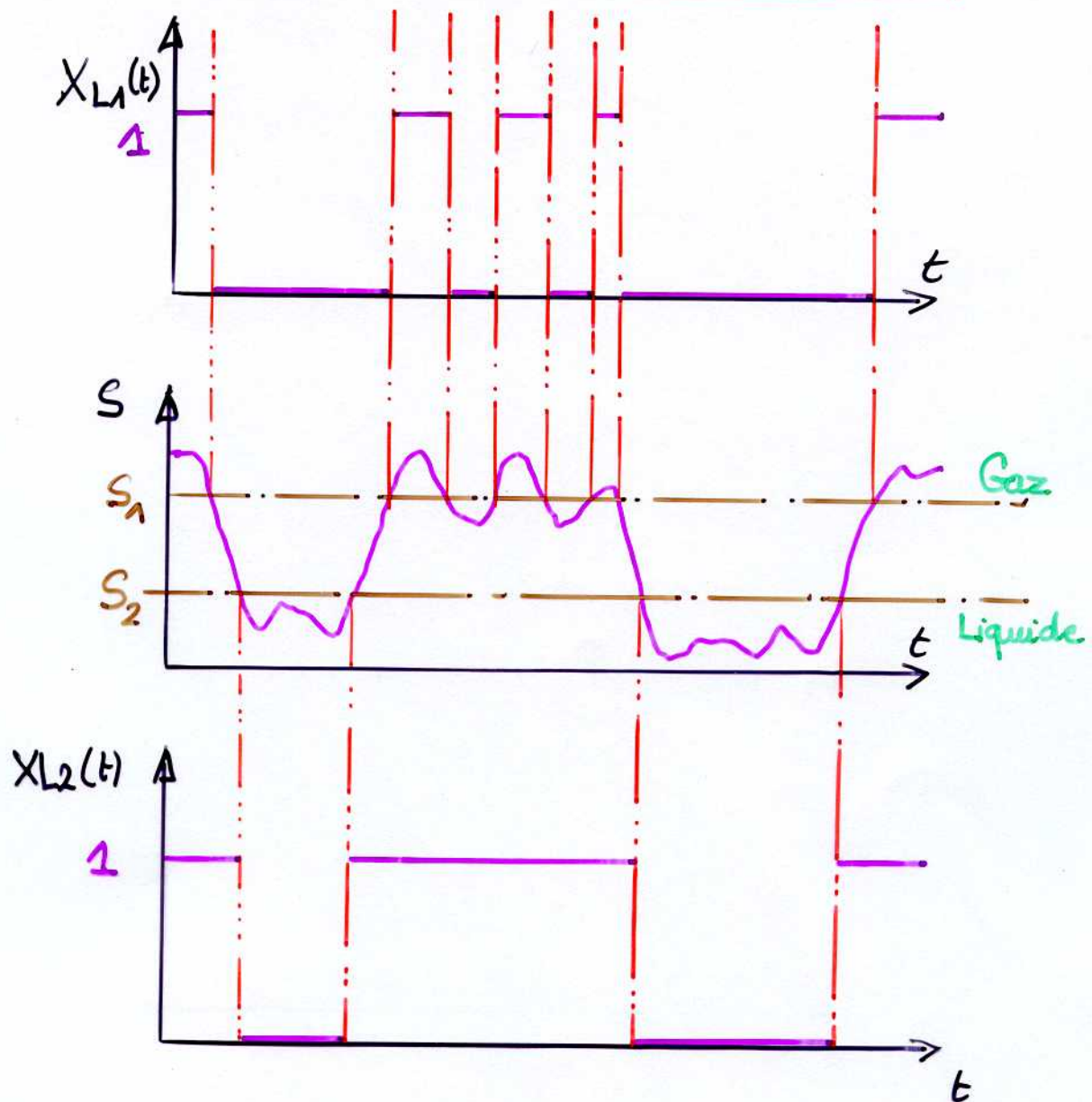
- Sondes optiques (indice optique)
 eau
freon
 $T < 110^\circ\text{C}$



Pointe de la fibre



DETERMINATION des SEUILS



- Détermination du seuil s . $\alpha_1 < \alpha_2$!

Méthode de référence ex $\Delta P \rightarrow \overline{R}_G$

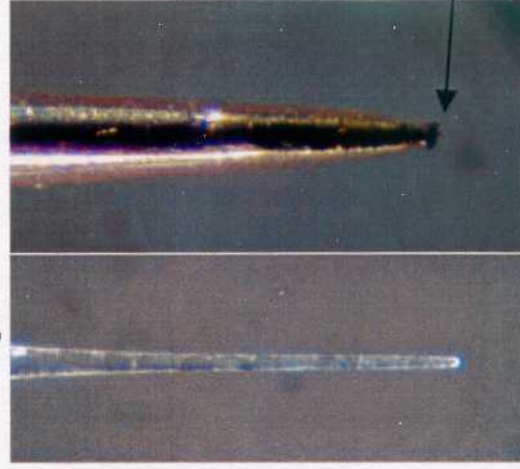
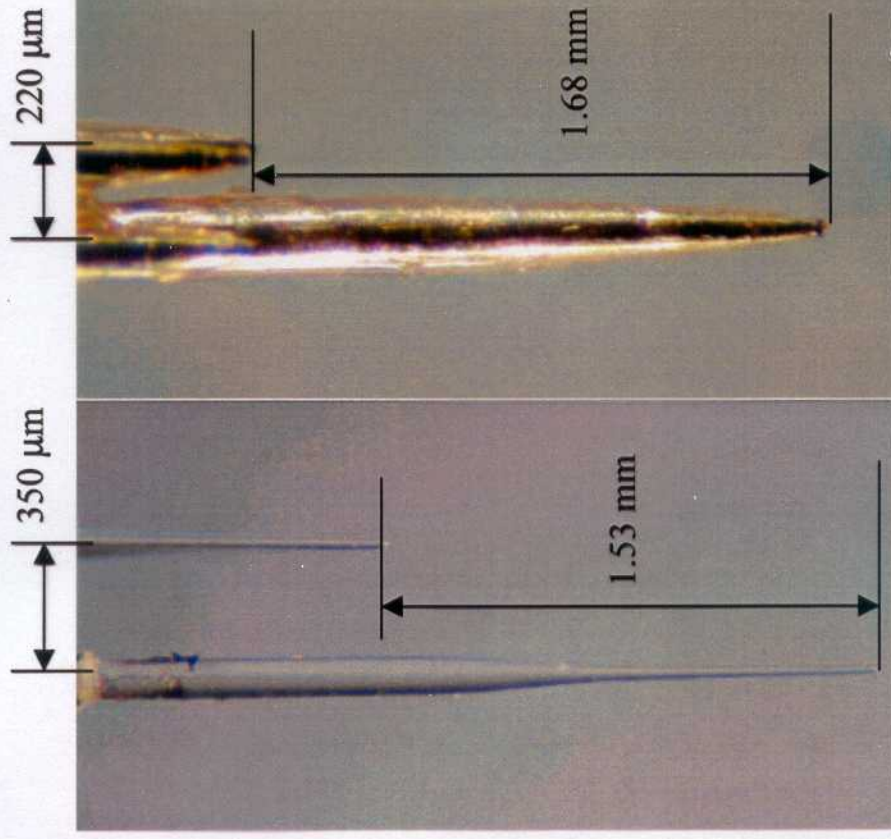
$$\alpha_G(s) = \frac{T_G(s)}{T} \quad ; \quad \text{profil ds une conduite}$$

$$\text{Ajustement } s \quad / \quad \langle \alpha_G(s) \rangle = \overline{R}_G$$

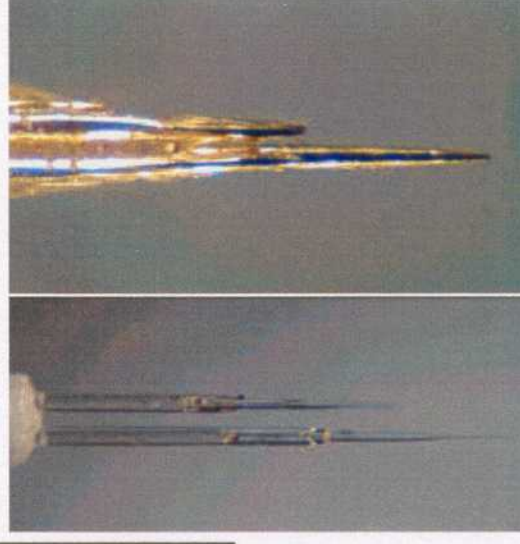
Optical vs Electrical four-sensor probe

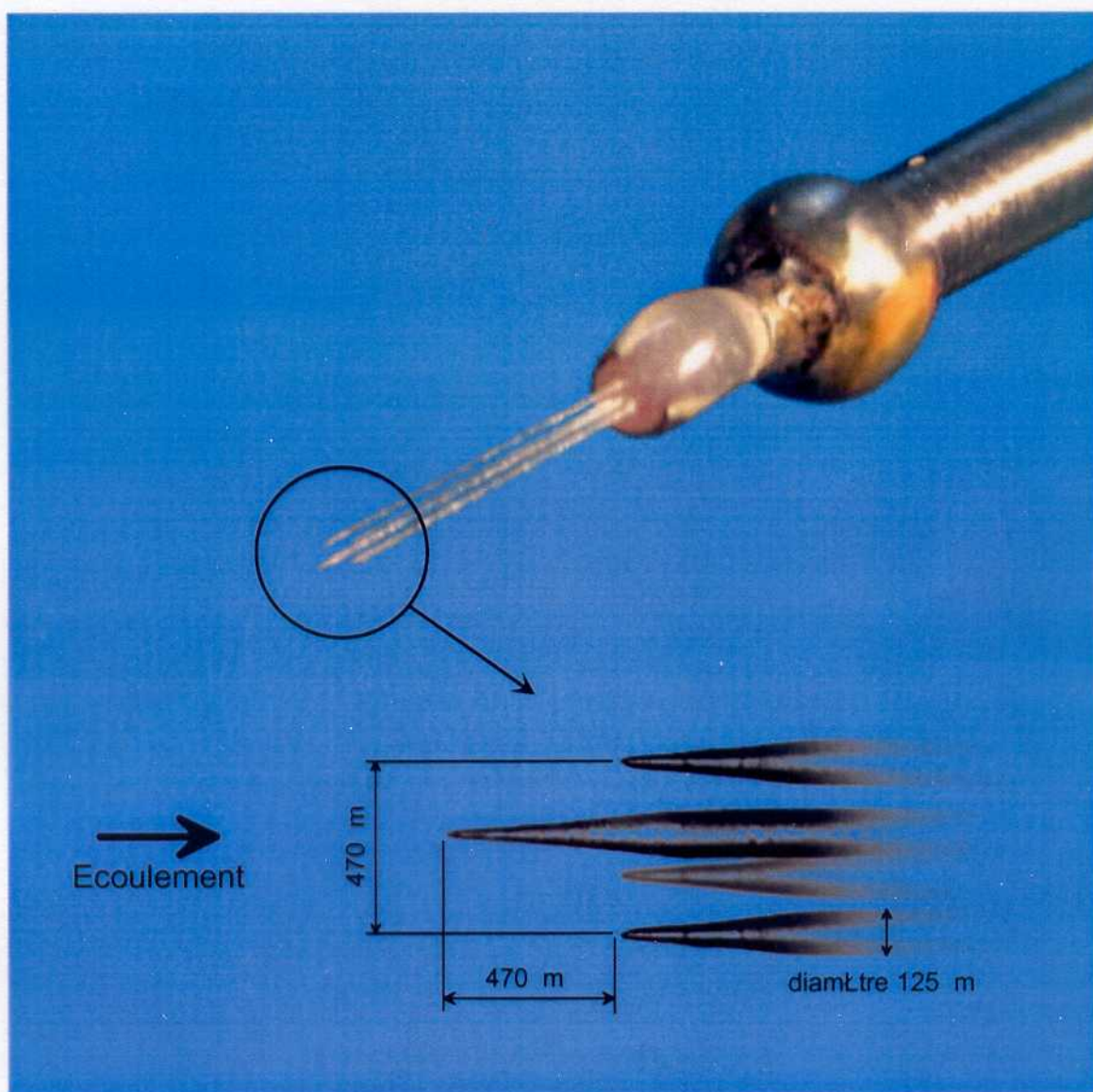
$$\phi_{\text{cond}} \approx 200 \mu\text{m}$$

$$\phi_{\text{opt}} \approx 50 \mu\text{m}$$



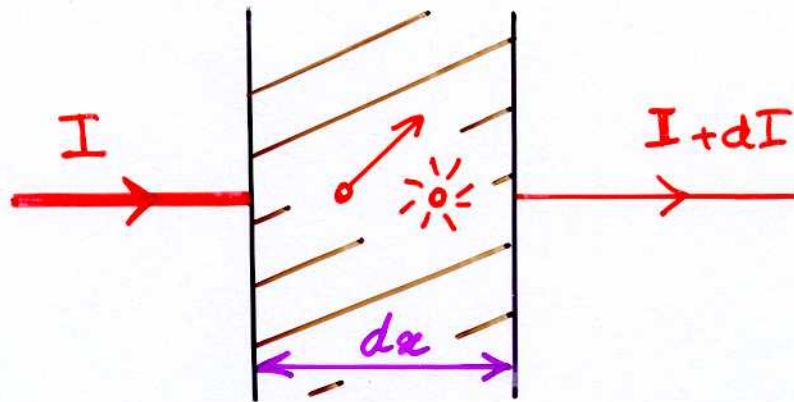
Overall view





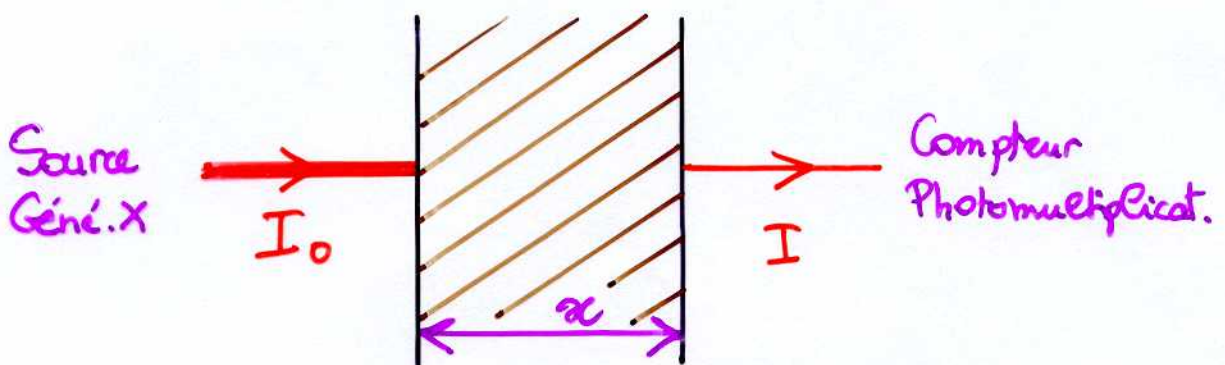
ATTENUATION RX en γ

Méthodes Photoniques



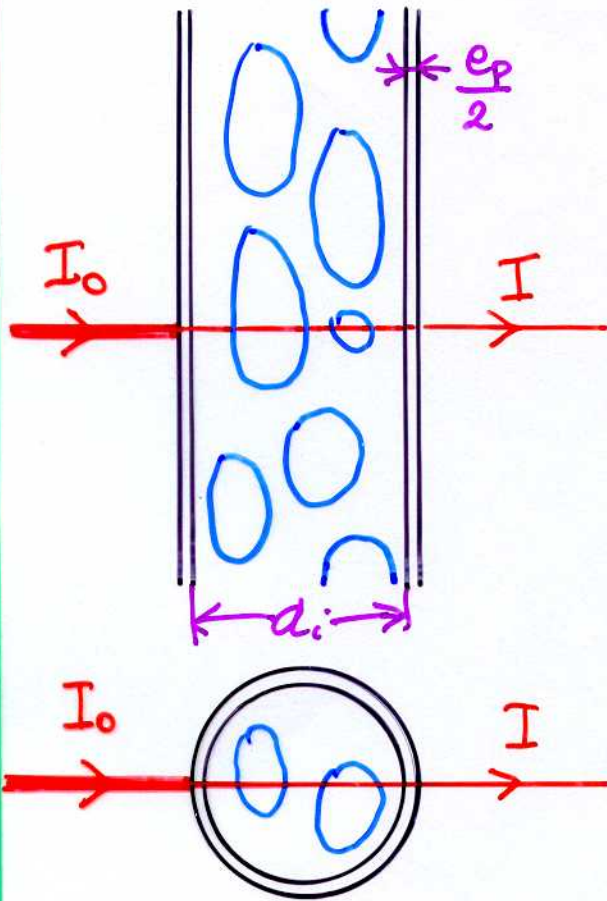
loi de Beer-Lambert :
faisceau collimaté, monoenergie (raie)

$$dI = -\mu I dx \quad [\mu] = L^{-1}$$



$$I = I_0 \exp(-\mu x) = I_0 \exp\left(-\left(\frac{\mu}{\rho}\right) \rho x\right)$$

FRACTION LINÉIQUE de Phase



- Loi de Beer Lambert

$$I = I_0 \exp(-\mu_p e_p) \cdot \exp(-\mu_L (1-R_{G1}) d_i) \cdot \exp(-\mu_G R_{G1} d_i)$$

$$R_{G1}(z, t) \triangleq \frac{L_G}{L_G + L_L} = \frac{L_G}{d_i}$$

- Approximation Base Fraction

Monophasique Gaz: $I_G = I_0 \exp(-\mu_p e_p)$

Monophasique Liquide: $I_L = I_0 \exp(-\mu_p e_p) \exp(-\mu_L d_i)$

Ecoulement diphasique: $I = I_0 \exp(-\mu_p e_p) \exp(-\mu_L (1-R_{G1}) d_i)$

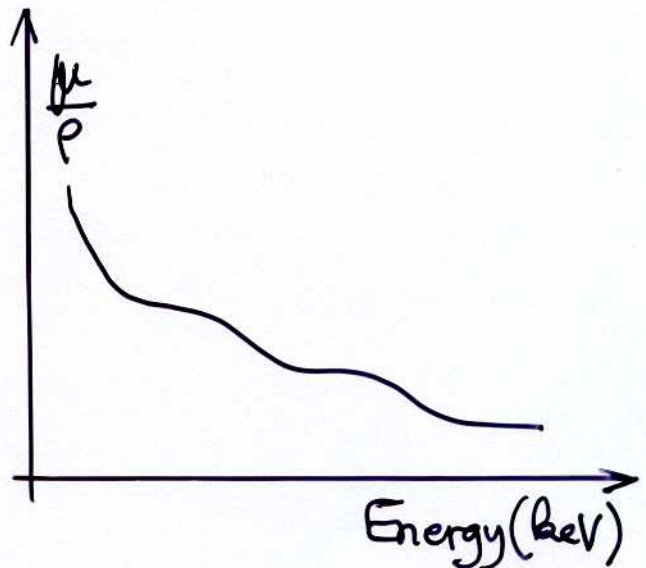
Elimination $I_0, \mu_p e_p \Rightarrow R_{G1} = \frac{\log I/I_L}{\log I_G/I_L}$

SOURCES D'ERREURS

- Contraste

$$\frac{I_G}{I_L} = \exp\left(\frac{\mu}{\rho_L}\right) \rho_L di$$

→ travailler à basse En



- Erreurs statistiques

$$I \propto N; \frac{\Delta N}{N} = \sqrt{\frac{1}{N}} \rightarrow \text{travailler à haute Energie}$$

- Fluctuations du taux de vide

$$\text{Mesure de } [I] = \int_0^z I dt \quad \text{et} \quad I(t) \propto \exp R_G(t)$$

$$\exp f \neq \exp \bar{f} \quad (\text{non linéarité!})$$

ex: écoulement à poches $\Delta R_G \sim 0,20$, agité $\Delta R_G \sim 0,05$

- Stabilité de la source

$I_0(t) \rightarrow$ méthode à faisceau de ref. $I \rightarrow \frac{I}{I_0}$

- Durcissement du spectre (f. multi-énergie)

filtres, Spectrométrie, étalonnage direct

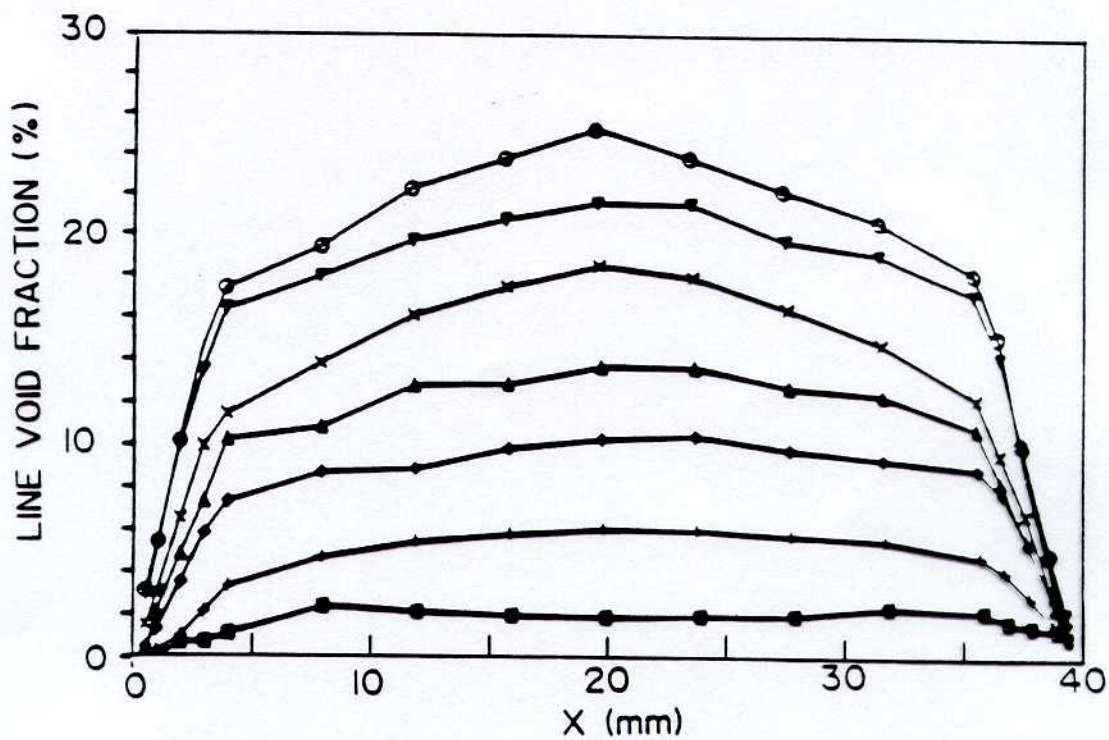


Figure 11 : Void fraction profiles for stagnant flow ($v_l = 0$, $L = 40$ mm),
 ($\square \epsilon = 0.01$; $+ \epsilon = 0.04$; $\diamond \epsilon = 0.07$; $\triangle \epsilon = 0.10$;
 $\times \epsilon = 0.13$; $\nabla \epsilon = 0.16$; $\circ \epsilon = 0.19$).

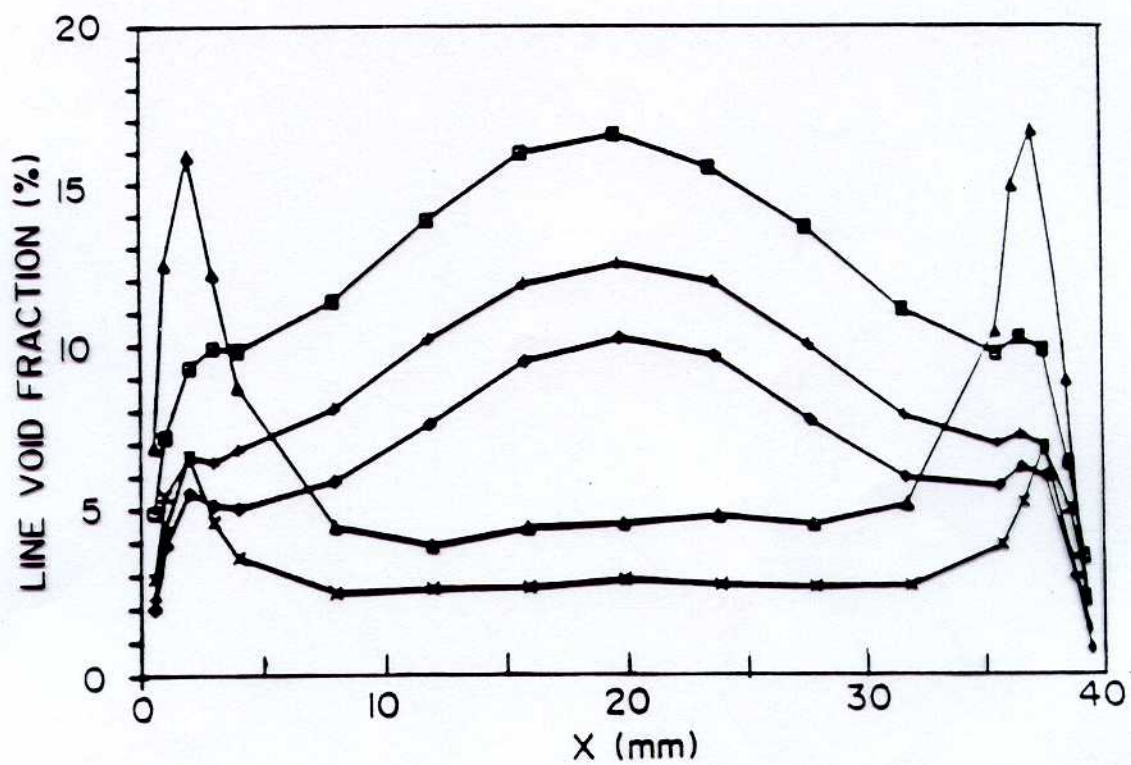


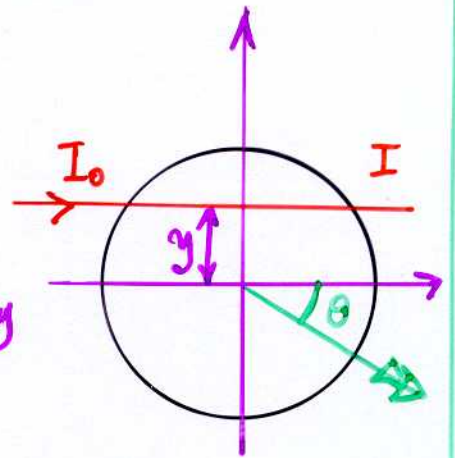
Figure 12 : Void fraction profiles for forced-circulation flow ($v_l = 2.0$ m/s, $L = 40$ mm), ($\times m_g = 0.1$ g/s, $\epsilon = 0.030$; $\triangle m_g = 0.3$ g/s, $\epsilon = 0.061$; $\diamond m_g = 0.4$ g/s, $\epsilon = 0.069$; $+ m_g = 0.5$ g/s, $\epsilon = 0.089$; $\square m_g = 0.7$ g/s, $\epsilon = 0.123$).

TAUX de VIDE SURFACIQUE.

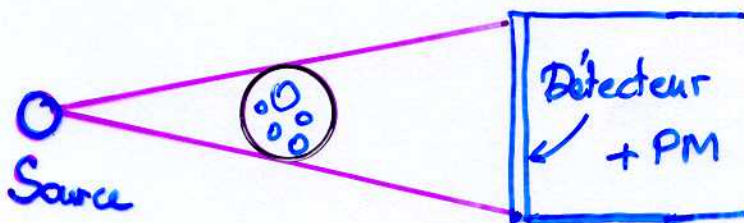
- Atténuation des Rayons X

- Moyenne spatiale $\overline{R_{G2}}$

$$\overline{R_{G2}} = \frac{1}{\pi R^2} \int_{-R}^R 2 R_{G1}(y) \sqrt{R^2 - y^2} dy$$



- Valeur Instantannée R_{G2}

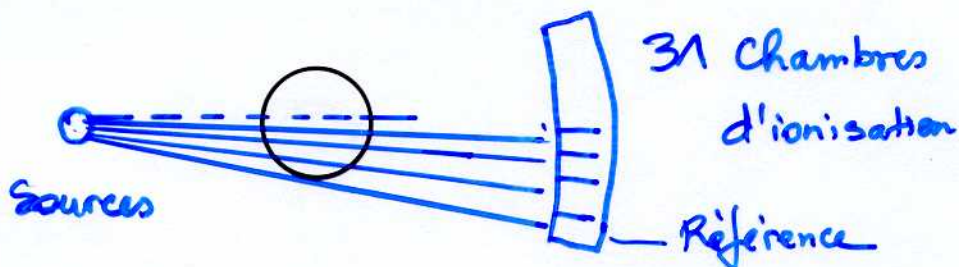


Limitations

$$\Delta R_{G2} \leq \pm 0,05$$

$$0 < R_{G2} < 0,8$$

- Densitomètre multifaisceaux (31)



- Tomographie à Rayons X

On mesure $\overline{R_{G1}}(y, \theta) \xrightarrow{\text{Inversion}} d_G(r, \theta)$

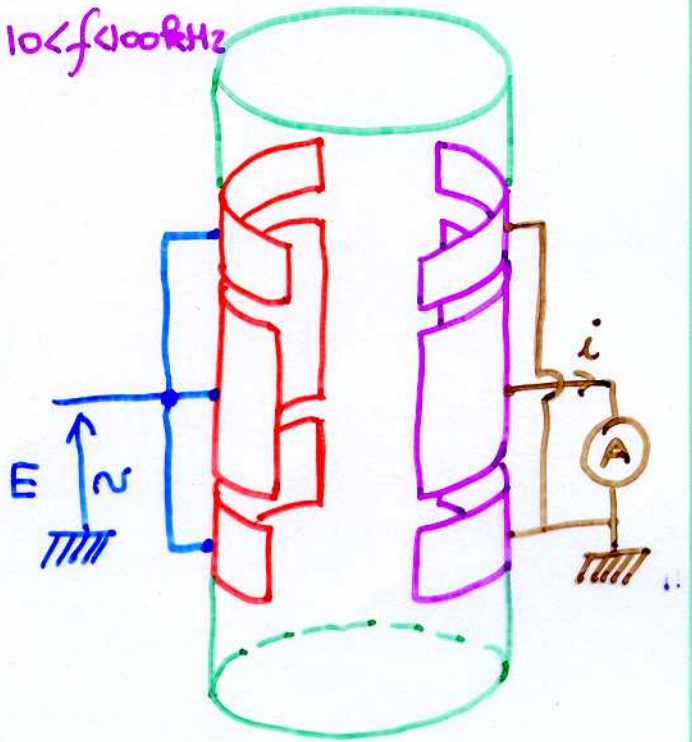
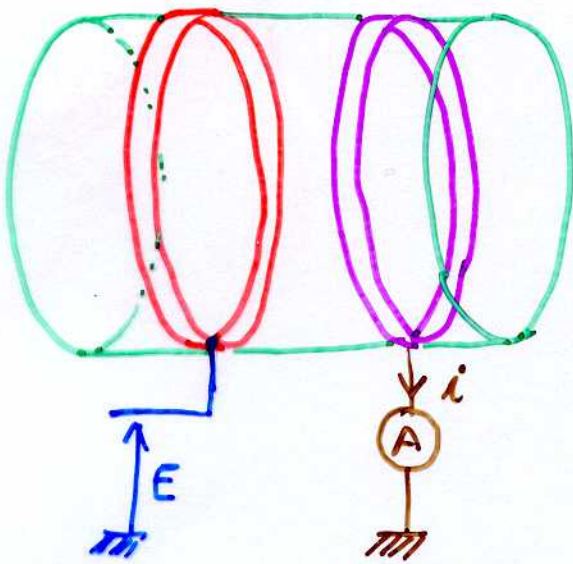
- Diffusion de neutrons

- traverse l'acier, diffusés + par l'eau.

DENSITOMETRES à IMPEDANCE

- Détermination impédance électrique 2ϕ

- Résistif : $\sigma_{2\phi}$
 - Capacitif : $\epsilon_{2\phi}$
- $10 < f < 1000 \text{ Hz}$



- Excitation $E \sim$ signal i

$$I = DE \sigma_c(T, c_1, c_2 \dots) f(R_{G2}, \dots)$$

- $R_{G3} \sim R_{G2}$ evolution lente ; $R_{G2}(t)$
- Méthode de référence $I_0 = E \sigma_c(T, c_1, c_2) f_0$
- Etalonnage / Modèle mtl. $f(R_{G2}, \dots)$
- Optimisation géométrique $f(R_{G2}, X)$

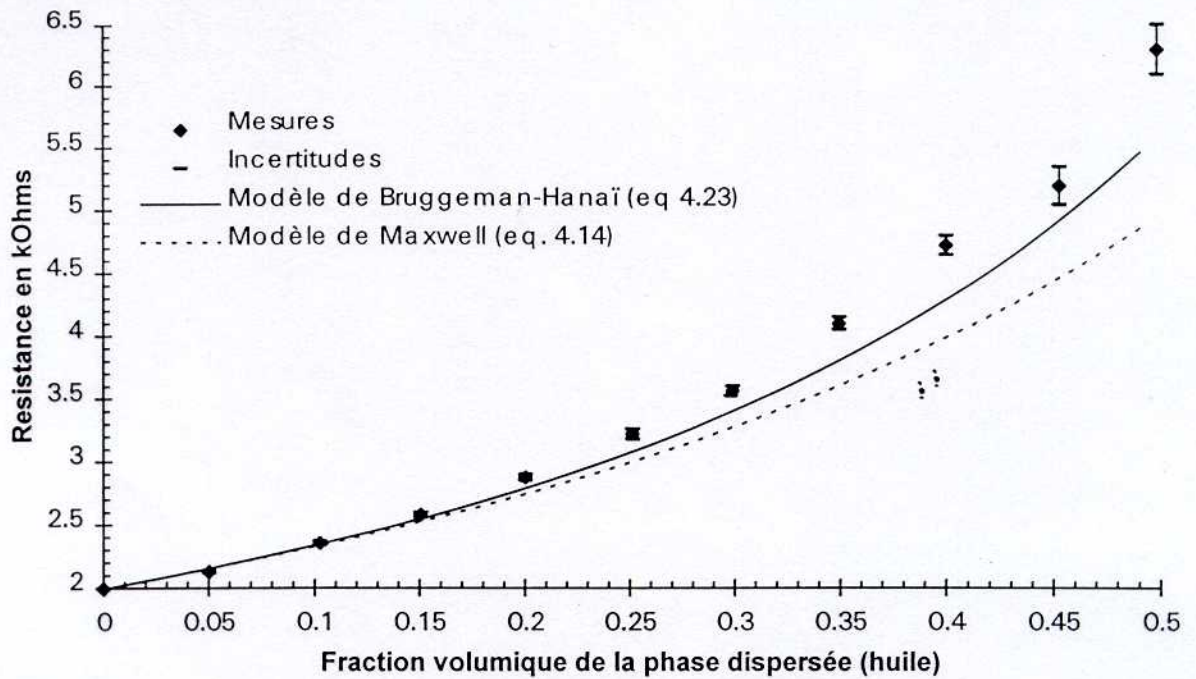


Figure 4.9 : Tracé de la résistance mesurée et calculée pour un écoulement eau-huile, lorsque la vitesse moyenne de l'écoulement est 56,6 cm/s et la température $18^{\circ}\text{C} \pm 0,3$.

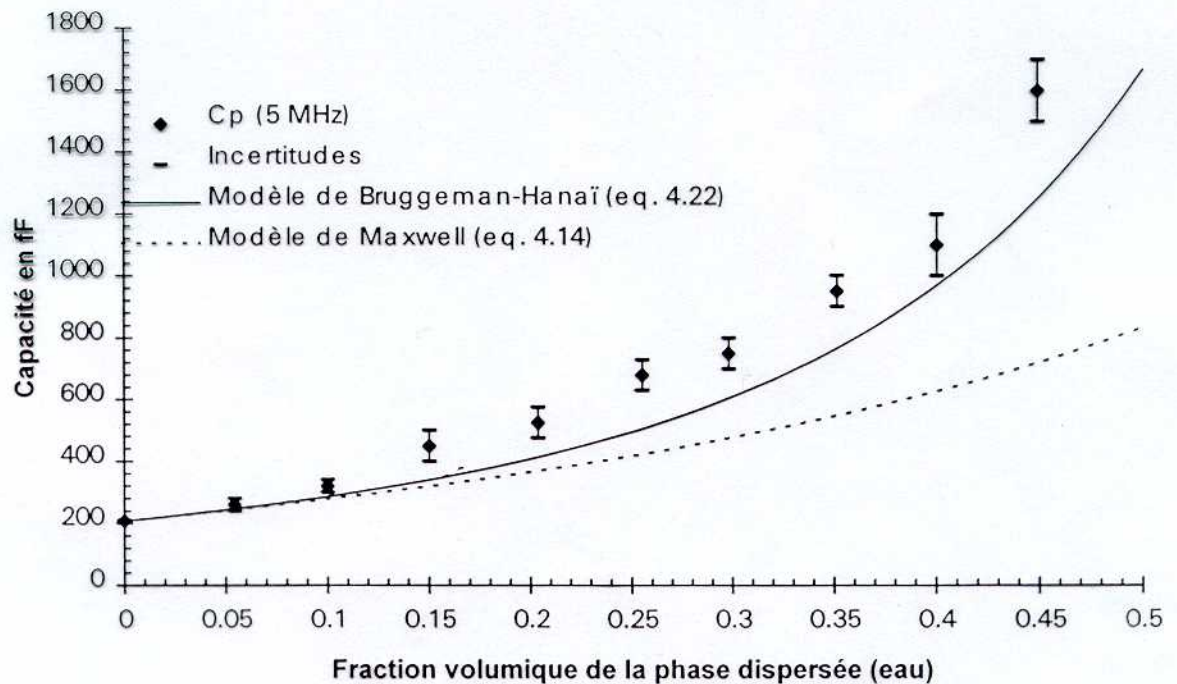
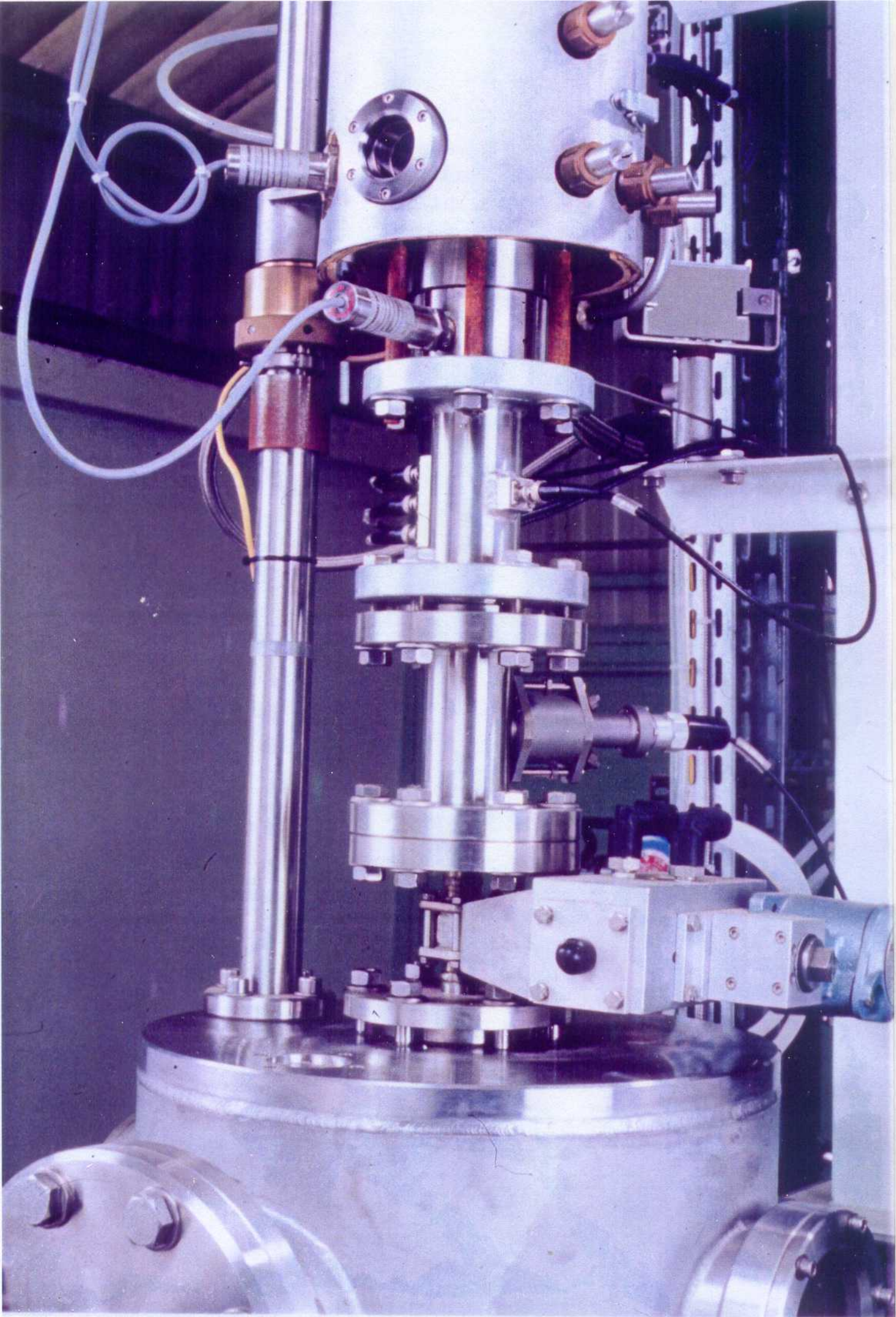
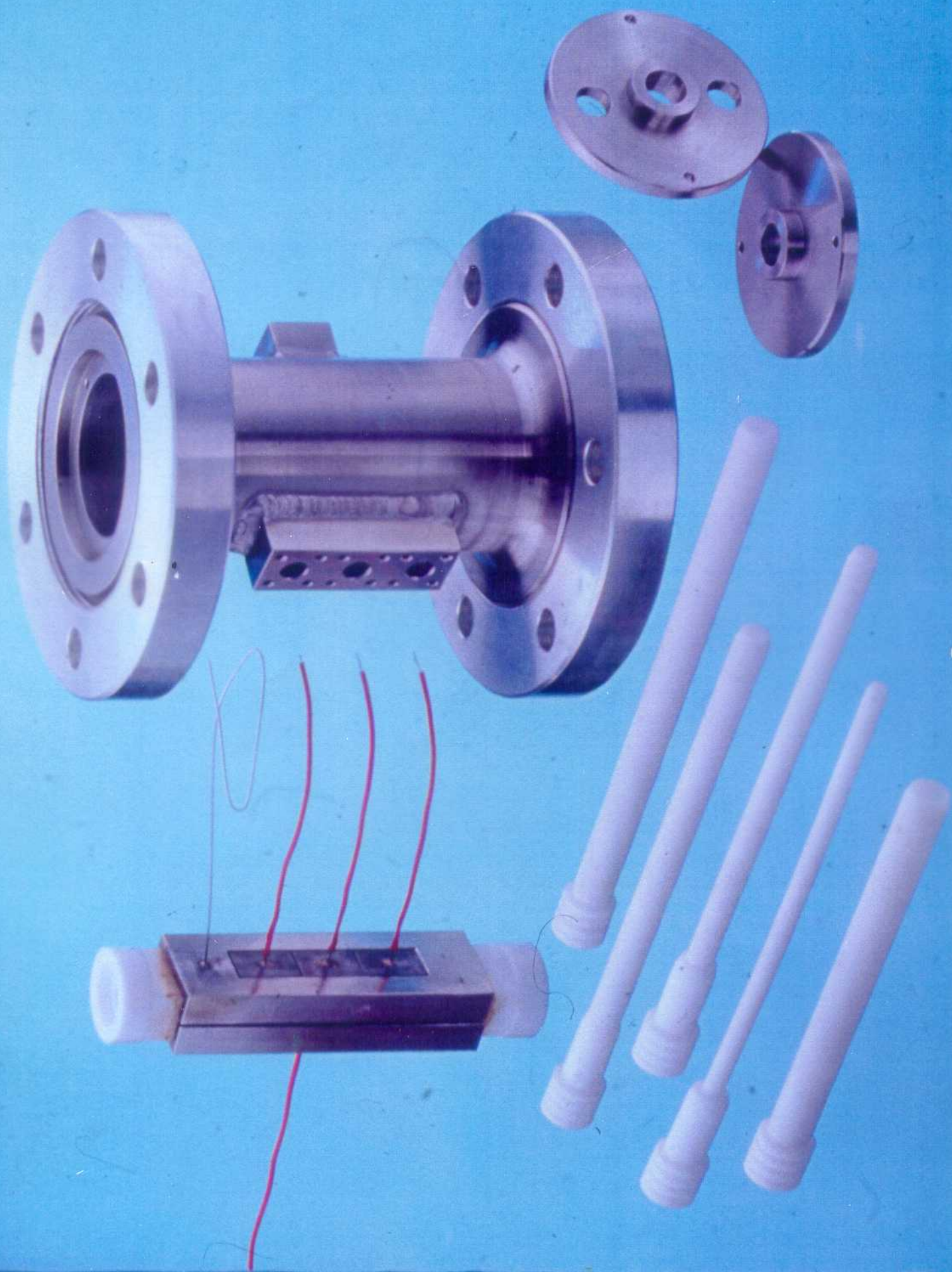


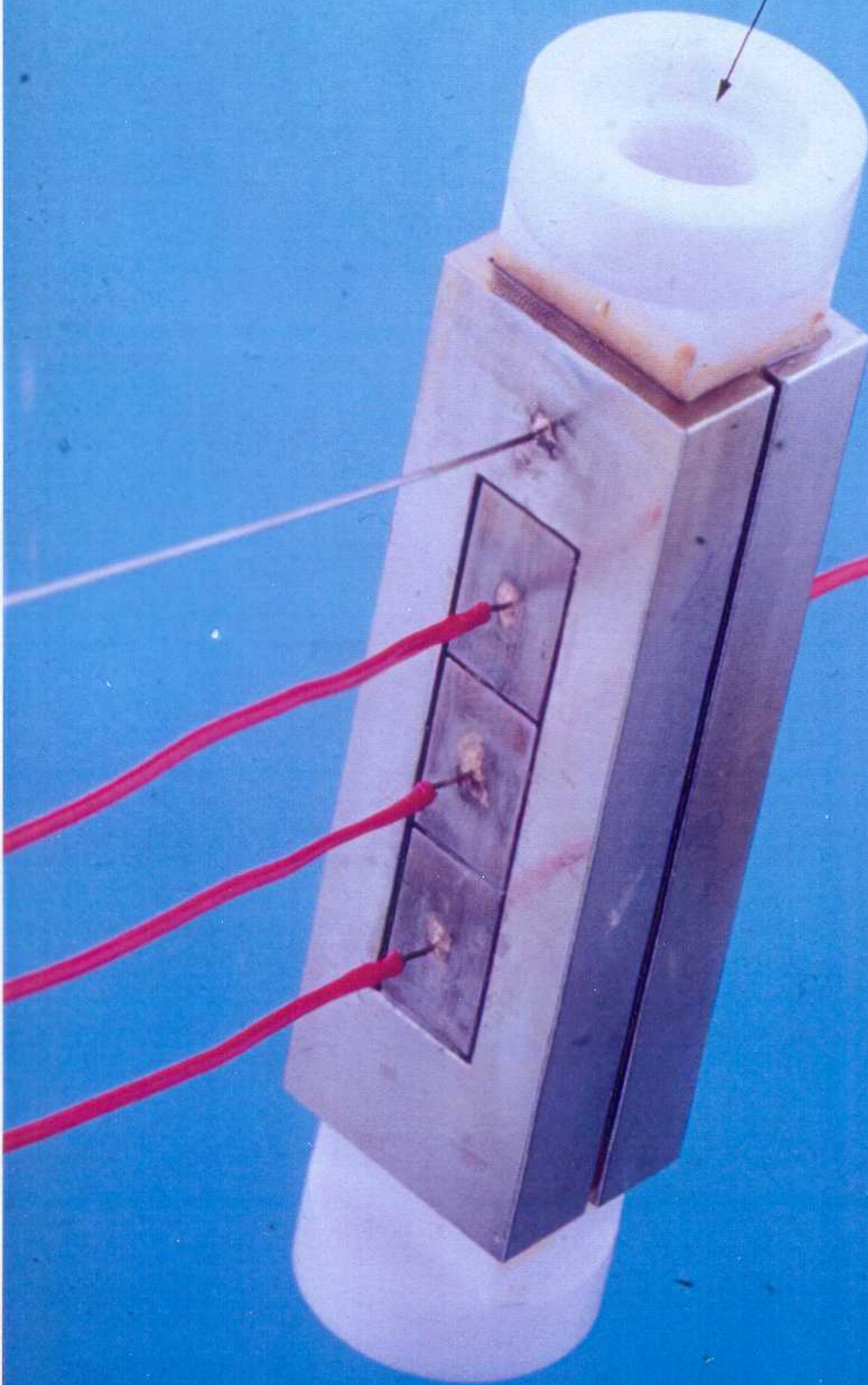
Figure 4.10 : Tracé de la capacité mesurée et calculée en écoulement huile-eau, lorsque la vitesse moyenne est de 56,6 cm/s et la température de $19,5^{\circ}\text{C} \pm 0,1$.

d'après Boyer, C. (1996)



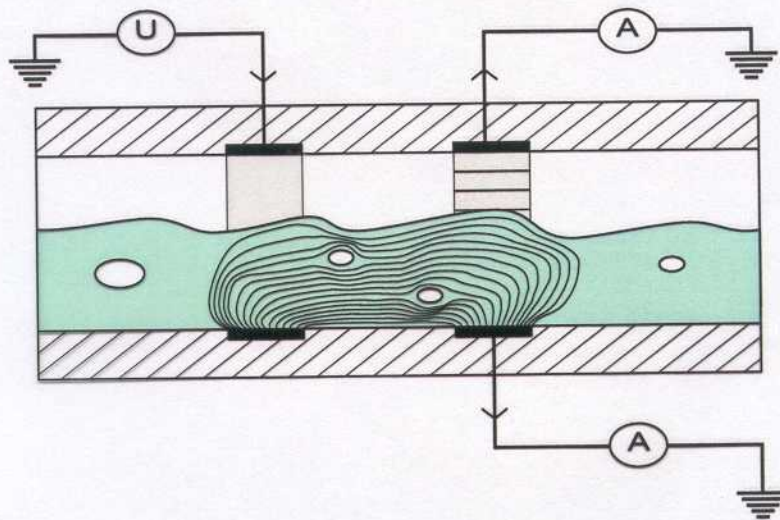


Ø INTERIEUR
12 mm



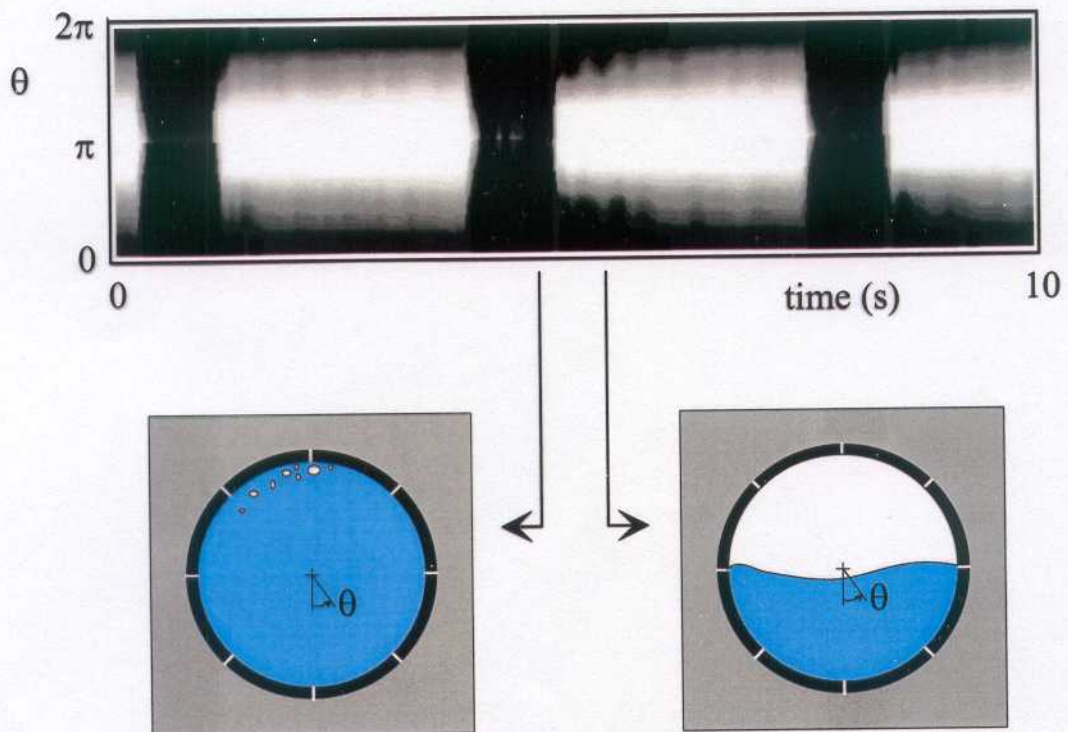
Impedance probe

non intrusive
spatial and dynamic information
easy optimization (3D simulation)

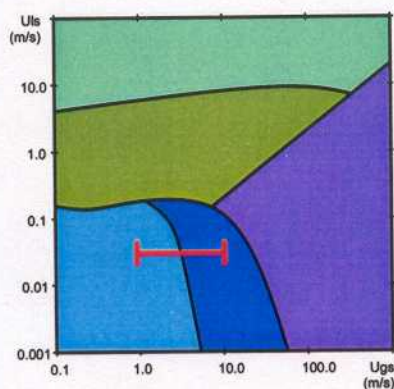


Flow monitoring (real time)

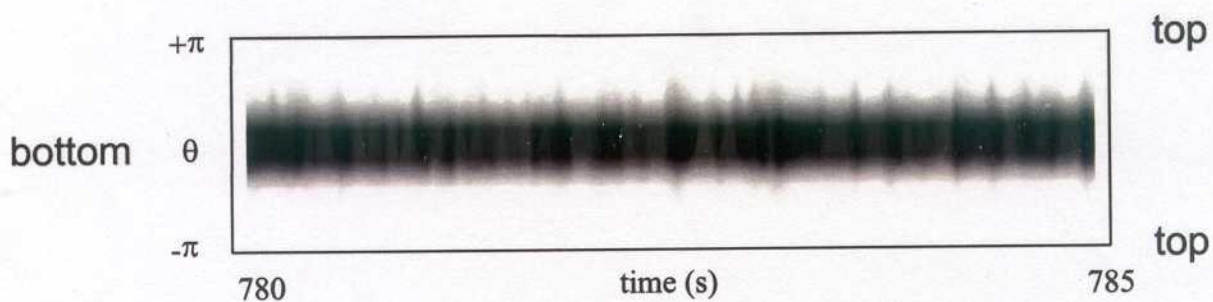
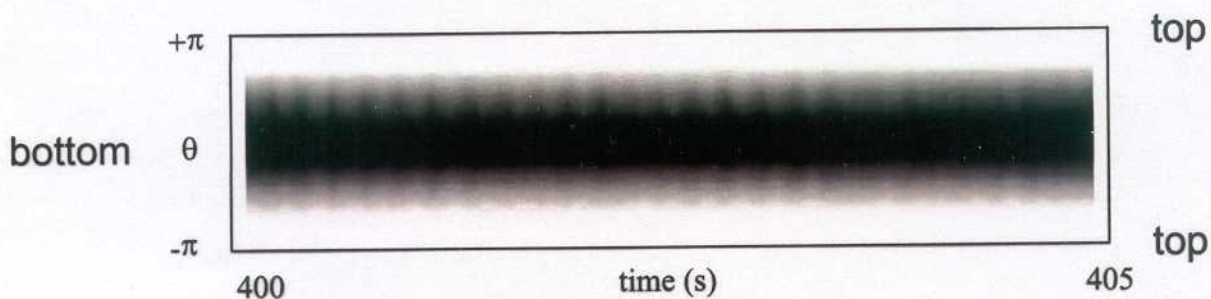
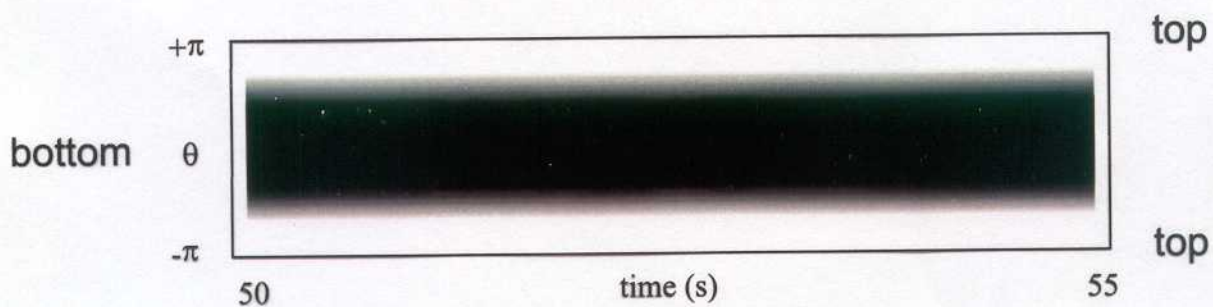
Slug flow



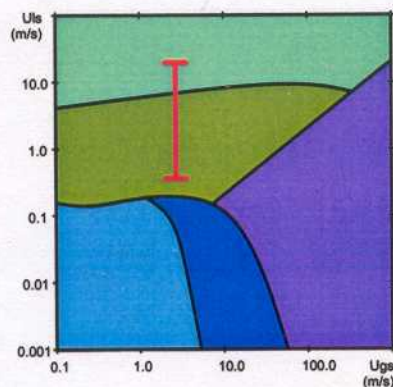
Smooth - Rugged Stratified Flow Evolution



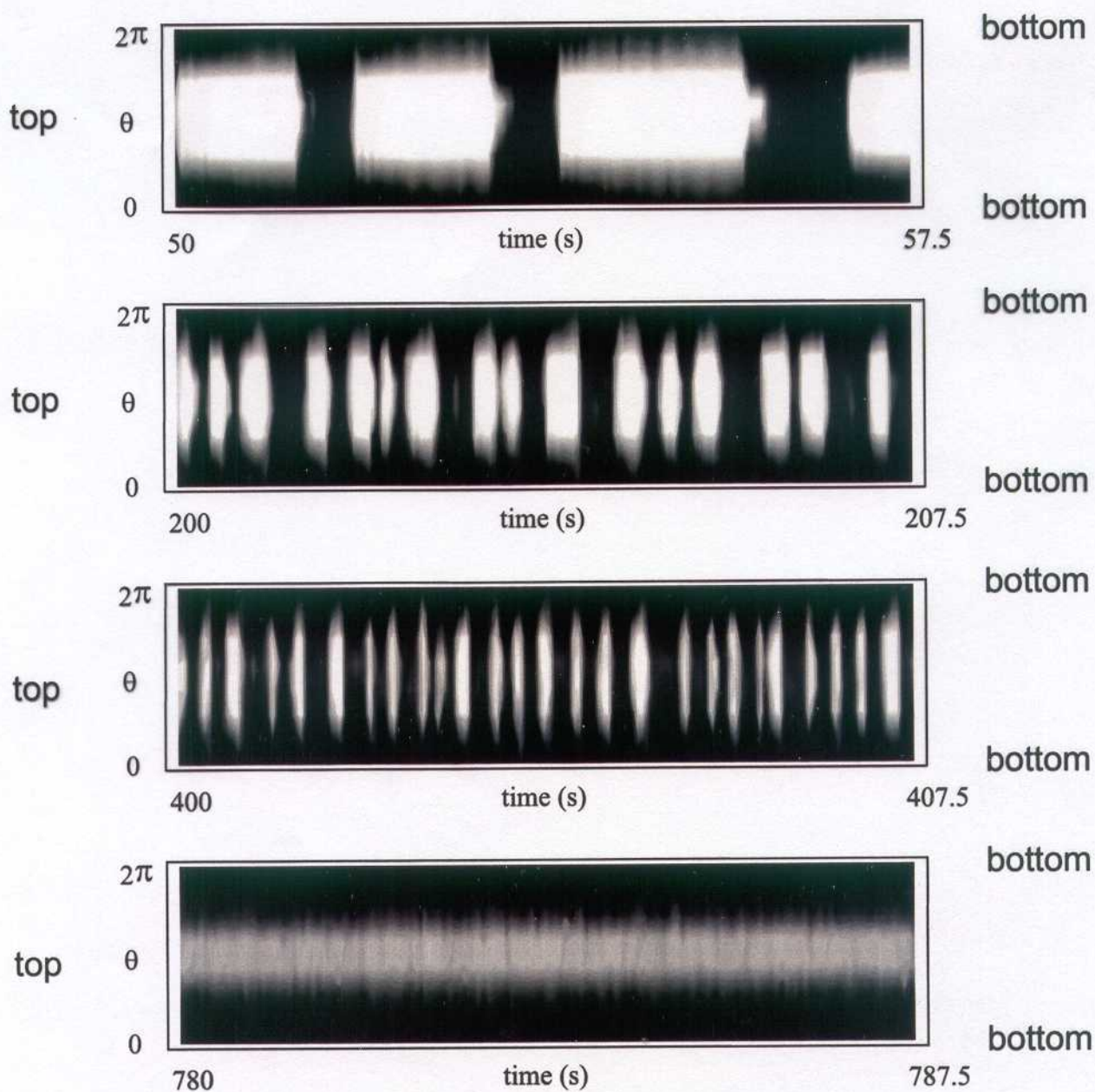
$$Q_{\text{water}} = 0.5 \text{ m}^3/\text{h} - Q_{\text{air}} = 10 - 60 \text{ m}^3/\text{h}$$



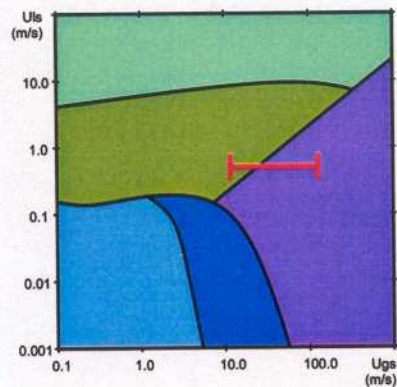
Intermittent - Bubbly Flow Evolution



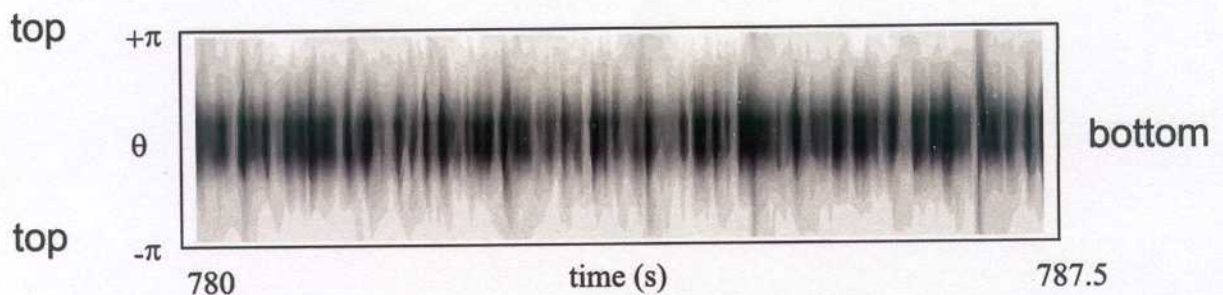
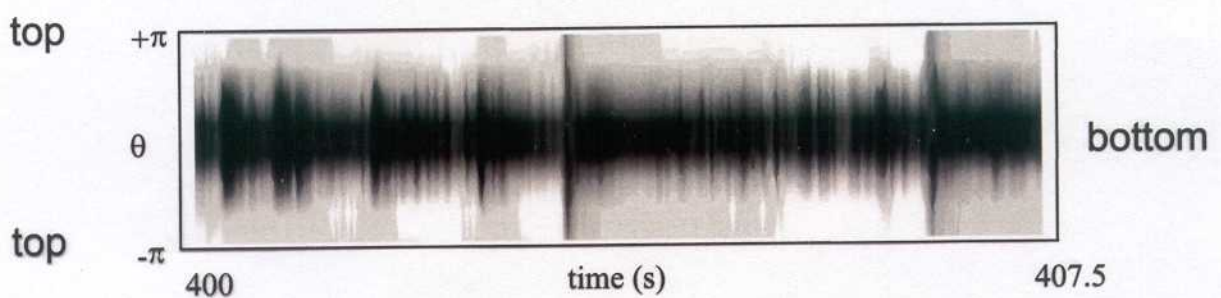
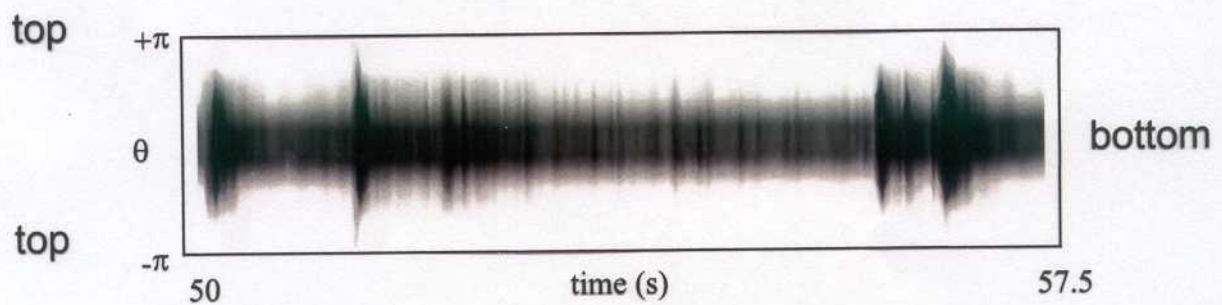
$$Q_{\text{air}} = 7.5 \text{ m}^3/\text{h} - Q_{\text{water}} = 5 - 50 \text{ m}^3/\text{h}$$



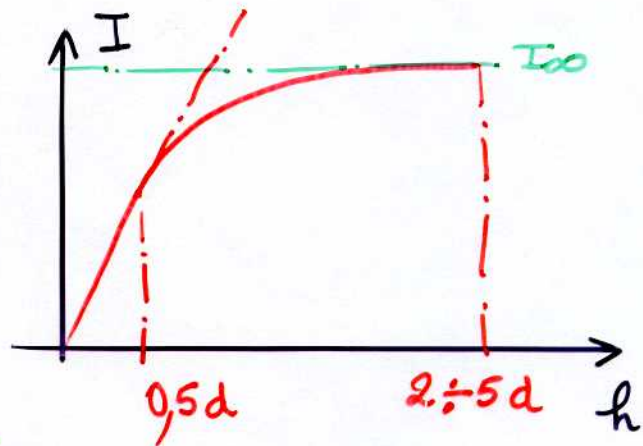
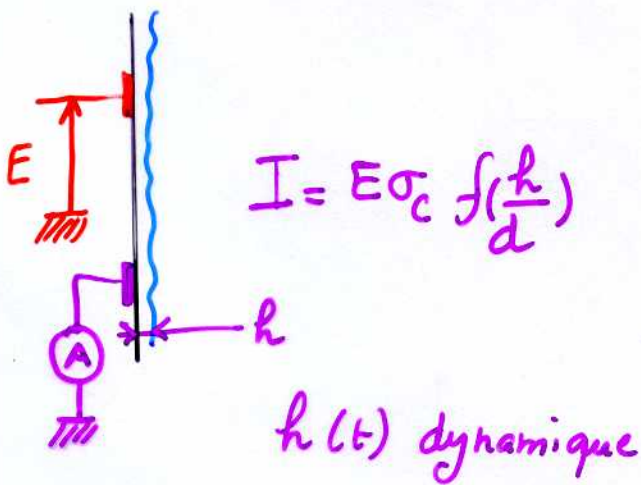
Intermittent - Annular Flow Evolution



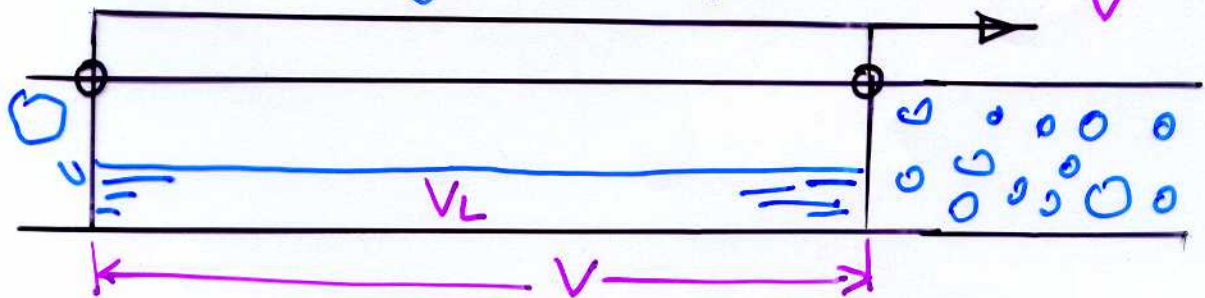
$$Q_{\text{water}} = 15 \text{ m}^3/\text{h} - Q_{\text{air}} = 70 - 250 \text{ m}^3/\text{h}$$



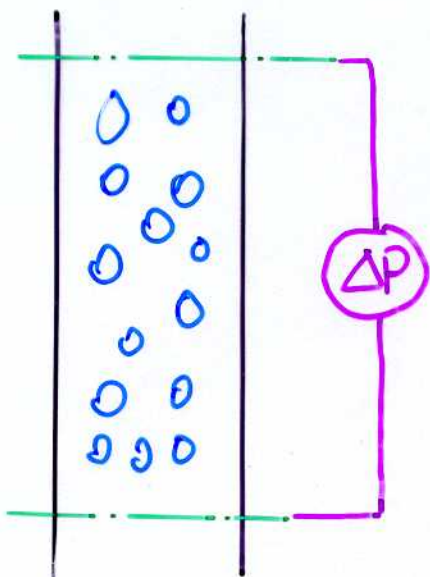
- Epaisseur de film liquide



- Vannes à fermeture rapide : $R_{L3} = \frac{V_L}{V}$



- Méthode gravitaire



- Vitesses "faibles"
 $\Rightarrow$ frottement et accélération $\rightarrow 0$

$$\Delta P = \rho_m g h$$

$$\rho_m = (1 - R_{G3}) \rho_L + R_{G3} \rho_G$$

MASS FLOW RATE DETERMINATION
IN TWO-PHASE FLOW;
THE GRENOBLE EXPERIENCE

- EQUIPMENT
- FLOW RATE CALCULATION

EQUIPMENT

• Gammadensitometer $\rightarrow R_{GI}$

• Venturi

$$(1) \quad M_v = k \sqrt{\rho_v \Delta p}$$

k (in m^2) : coefficient obtained from a calibration in liquid single phase flow

M_v (in kg/s) ρ_v (in kg/m^3) Δp (in Pa)

$$(2) \quad \frac{1}{\rho_v} = \frac{x^2}{R_v \rho_G} + \frac{(1-x)^2}{(1-R_v) \rho_L}$$

• Turbine

$$(3) \quad M_T = \lambda \rho_T f$$

λ (in m^3) : coefficient obtained from a calibration in liquid single phase flow

M_T (in kg/s) ρ_T (in kg/m^3) f (in s^{-1})

$$(4) \quad \frac{1}{\rho_T} = \frac{x^2}{R_T \rho_G} + \frac{(1-x)^2}{(1-R_T) \rho_L} \quad (\text{Popper, 1961; Rouhani, 1969})$$

Method #1: "Homogeneous model"

• Equipment:

Gamma densitometer	→	R_{G1}
Venturi	→	Δp
Turbine	→	f

• Hypotheses:

A. $R_{G2} \equiv R_{G1}$

B. $R_V \equiv R_T \equiv R_{G2}$

C. Homogeneous model

• Flow rate calculation: B: (2) & (4) → $\rho_V \equiv \rho_T$

$$\rho_V \equiv \rho_T = R_{G2} \rho_G + (1 - R_{G2}) \rho_L$$

(A) → $\rho_V \equiv \rho_T = R_{G1} \rho_G + (1 - R_{G1}) \rho_L$

(1) → M_V (3) → M_T

$$C \rightarrow \frac{1}{\rho_V} \equiv \frac{1}{\rho_T} \equiv \frac{x}{\rho_G} + \frac{(1-x)}{\rho_L} \rightarrow x$$

• Consistency check: $M_V \equiv M_T$

• Application:

Rod bundle blowdown (Ω)

Method #2: "Venturi - turbine"

- Equipment:

$$\begin{array}{lcl} \text{Venturi} & \longrightarrow & \Delta p \\ \text{Turbine} & \longrightarrow & f \end{array}$$

- Hypotheses:

$$A. \rho_v \equiv \rho_T \hat{=} \rho$$

- Flow rate calculation:

$$M_v \equiv M_T \hat{=} M$$

$$(1) \& (3) \longrightarrow M = \frac{R^2 \Delta p}{\lambda f}$$

$$\text{If } B. \text{ Homogeneous model} \longrightarrow \frac{1}{\rho} = \frac{x}{\rho_G} + \frac{(1-x)}{\rho_L}$$

- Application:

Rod bundle blowdown (Ω)

TURBINE FLOWMETERS

DIAMETER :

12 TO 154 MM

BLADE MATERIAL :

STAINLESS STEEL
TITANIUM

MAXIMUM ROTATIONAL SPEED :

12 MM DIA : 120 000 RPM
154 MM DIA : 15 000 RPM

MAXIMUM ROTATIONAL ACCELERATION :

600 000 REVOLUTIONS/S²

MAXIMUM OPERATING TEMPERATURE :

780°C
900°C FOR SHORT PERIODS

DYNAMIC RANGE :

0.05 TO 225 M/S (WATER ; 46 MM DIA)

WASH	36	Measurement of the INTEGRAL specific area $\Gamma(t)$ in pipe flows
	26	

Methods compared:

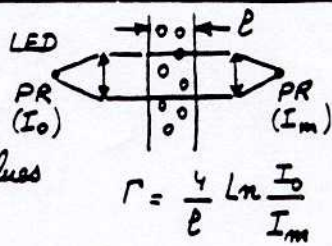
- photographic (bubble diameter measurements)
- chemical (oxidation of sodium sulfite into sodium sulfate)
- optical (light attenuation)

WASH	36	Photographic method
	27	

- Advantages:
 - instantaneous picture
- Disadvantages:
 - limited to bubbly or droplet flow
 - 2D picture of the flow near the wall
 - very tedious

WASH	36	Chemical method
	28	

- Advantages:
 - does not depend on flow pattern
- Disadvantages:
 - limited to steady state flow
 - limited to well defined contactors
 - requires special fluids (aqueous solution of sodium sulfite, oxygen and Co^{++} as a catalyst)
 - very lengthy (concentration determination)
 - requires a calibration set-up

WASH	36 31	Optical technique
	- Advantages: - simple - gives instantaneous values - Disadvantages: - limited to bubbly or droplet flow - limited to homogeneous dispersions	 $r = \frac{4}{l} \ln \frac{I_0}{I_m}$

LED: light emitting diode

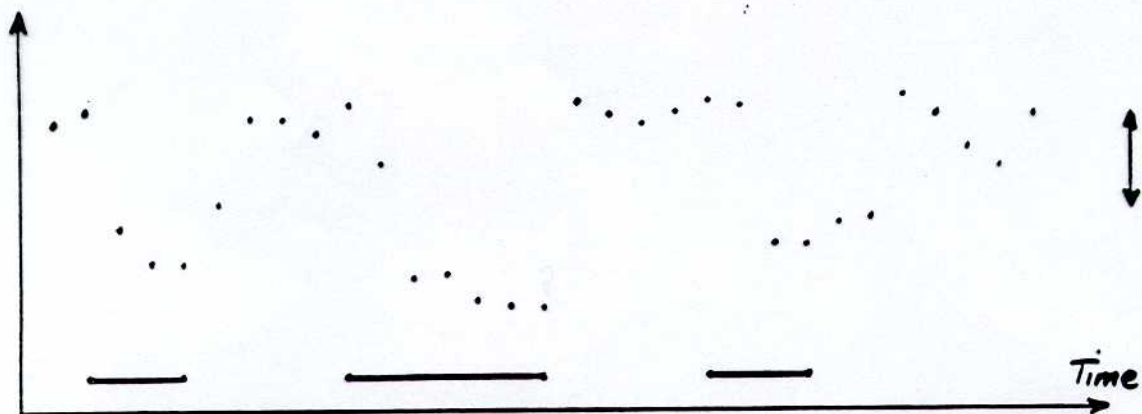
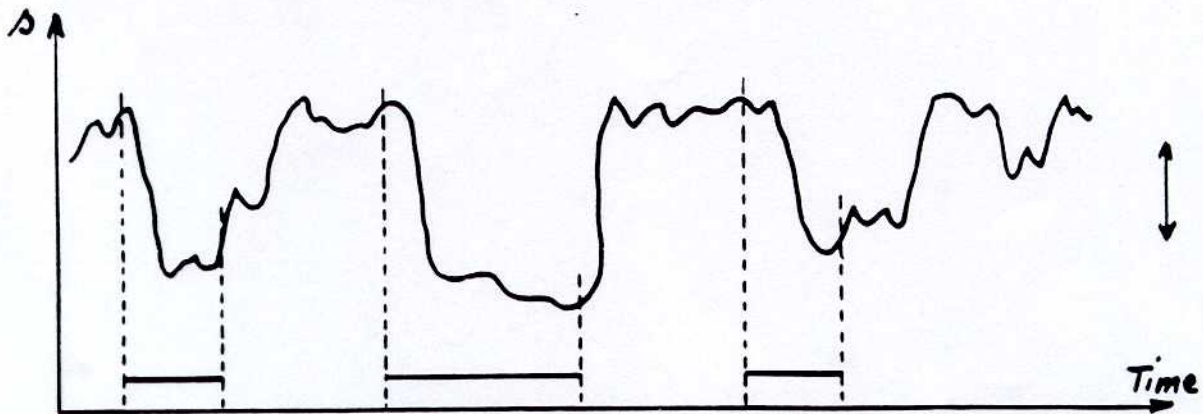
PR: photo receiver

I_0 : emitted intensity

I_m : received intensity

THERMAL ANEMOMETER SIGNAL PROCESSING

• AMPLITUDE DISCRIMINATION: RESCH

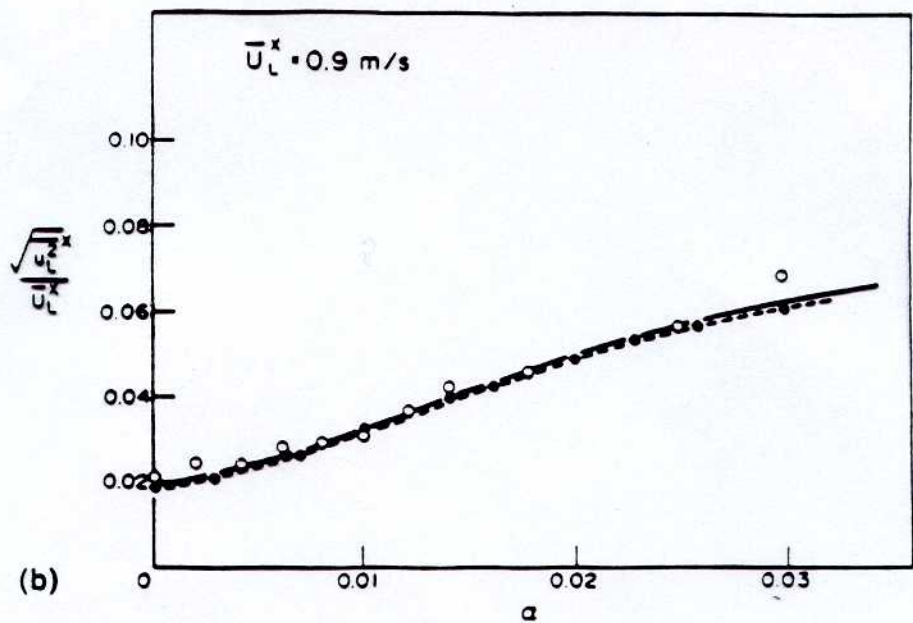
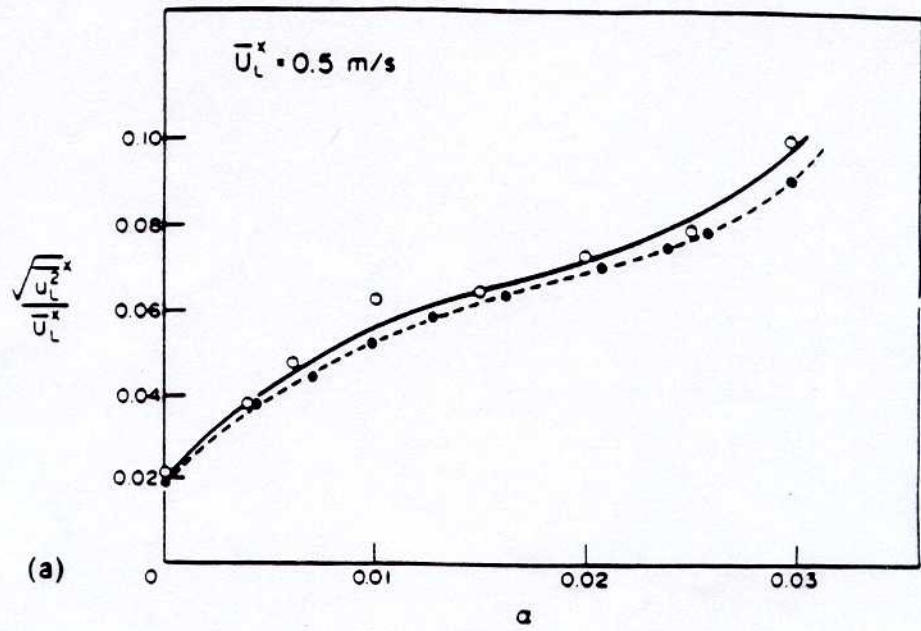


• OTHER TECHNIQUES :

- PDF splitting (DELHAYE, GALAUP, HERRINGE)
- Threshold on the signal (JONES)
- Threshold on the signal derivative (DUKLER)

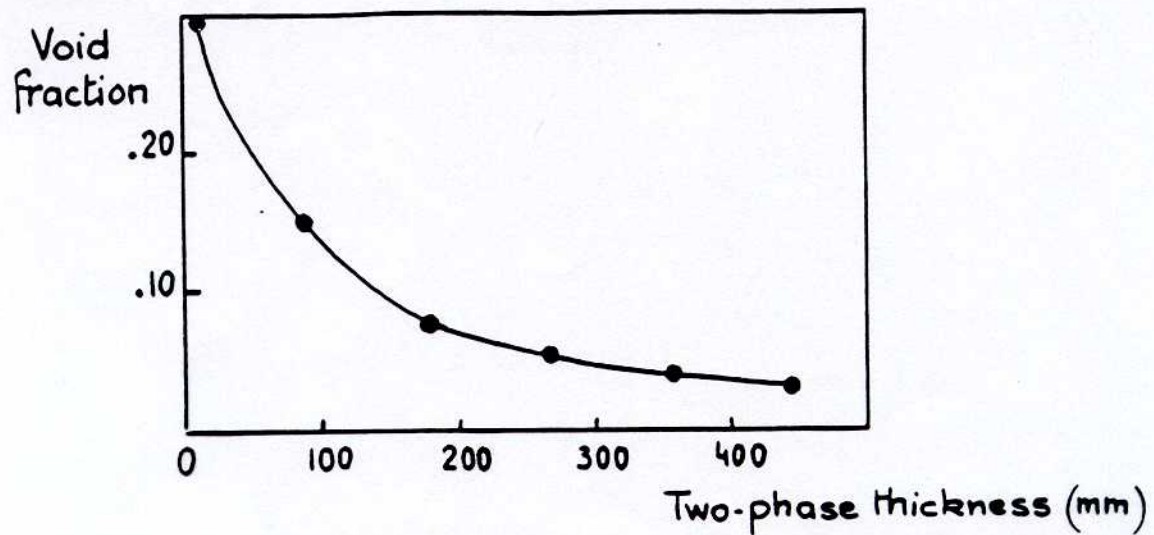
• GIVES :

- Local void fraction α_g
- Instantaneous liquid velocity w_L



MARIÉ, 1983

Limitations



See Marié & Lance (1984)

LES PREMIERS MODELES SIMPLE de TAUX de VIDE

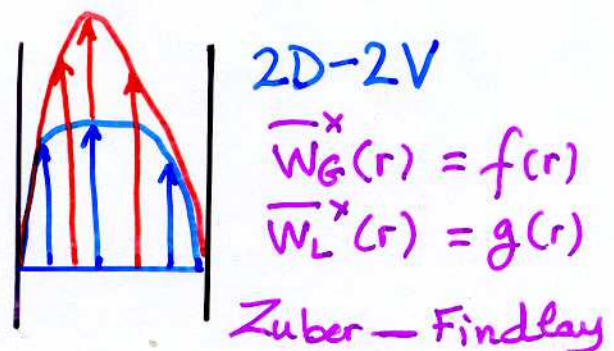
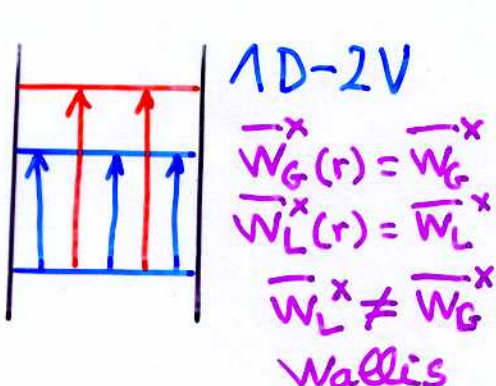
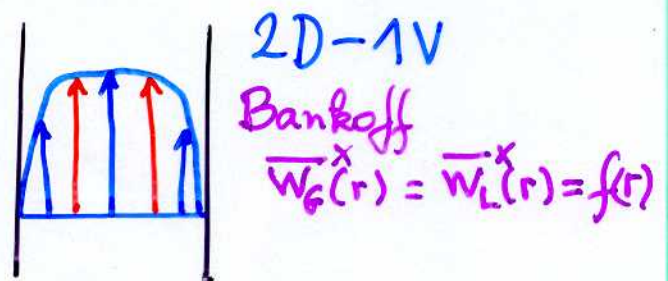
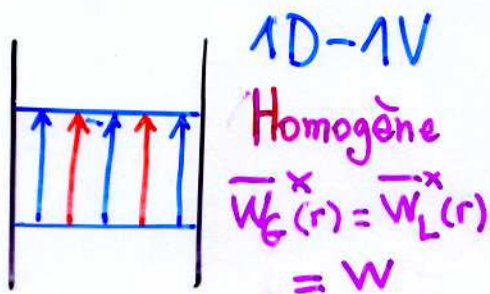
- Situations d'équilibre (établies)
transitoires lents.

– Idéalisation de l'écoulement.

- déséquilibre mécanique $\overline{w}_G^x \neq \overline{w}_L^x$
- déséquilibre thermodynamique
 $\overline{h}_L^x \neq h_{Lsat}(p)$
 $\overline{h}_V^x \neq h_{Vsat}(p)$

- Propriétés géométriques de l'écoulement
 – axisymétrie / profils radiaux...

déséquilibres imposés $\neq$ modèle non contraint



LE MODELE HOMOGENE.

• 1D-1V : $\overline{w_L^x} = \overline{w_G^x} = \text{cte}$

On se donne les débits : $\overline{Q_G}$ et $\overline{Q_L}$
 On calcule : $\overline{R_G}$ ou α

Etablissement commun aux 4 modèles

$$\overline{Q_G} \triangleq \int_{A_G} w_G dA \equiv \overline{A_G \langle w_G \rangle} = A \overline{R_G \langle w_G \rangle}$$

(CDM): Commutativité des moyennes spatiales et temporelles

$$\overline{Q_G} = A \overline{R_G \langle w_G \rangle} = A \langle \alpha_G \overline{w_G^x} \rangle$$

or $\overline{w_G^x}$ est uniforme ds la section et CDM

$$\overline{Q_G} = A \langle \alpha_G \rangle \overline{w_G^x} = A \overline{R_G} \overline{w_G^x} \quad \triangle$$

de même pour le liquide :

$$\overline{Q_L} = A (1 - \overline{R_G}) \overline{w_L^x}$$

or les vitesses sont égales

$$\frac{\overline{Q_G}}{\overline{Q_L}} = \frac{\overline{R_G}}{1 - \overline{R_G}} \Rightarrow \overline{R_G} = \frac{Q_G}{Q_L + Q_G} = \frac{Q_G}{Q} \triangleq \beta$$

titre volumique

Modèle homogène

$$\overline{R_G} = \beta$$

LE MODELE de BANKOFF.

• 2D - 1 V
$$\begin{cases} \overline{W}_L^x = \overline{W}_G^x = W_c \left(\frac{y}{R} \right)^{\frac{1}{m}} \\ \alpha_G = \alpha_c \left(\frac{y}{R} \right)^{\frac{1}{n}} \end{cases}$$

On se donne les débits: $\overline{Q}_G$ et $\overline{Q}_L$

On calcule $\overline{R}_{G2} \equiv \alpha^2$

Definition des débits moyens

$$\overline{Q}_G = A \alpha \overline{W}_G^x = A f(W_c, \alpha_c, m, n)$$

$$\overline{Q}_L = A (1-\alpha) \overline{W}_L^x = A g(W_c, \alpha_c, m, n)$$

Calcul de

$$\alpha \overline{W}_L^x = \overline{W}_G^x = h(W_c, m)$$

$$\overline{R}_G = \alpha^2 = k(\alpha_c, n)$$

on élimine α_c et W_c

$$R_G = K \beta$$

$$K = \frac{2(m+n+mn)(m+n+2mn)}{(n+1)(2n+1)(m+1)(2m+1)}$$

$$K = 0,6 \div 1 \quad 2 \leq m, n \leq 7$$

Corrélation de Bankoff.

$$K = 0,71 + 0,00145 p \quad (p \text{ en bar})$$

MODELE de WALLIS

• 1D-2V $\begin{cases} \overline{w_L^x} \neq \overline{w_G^x} \text{ mais uniformes} \\ \alpha_G(r) = \alpha_G \quad '' \end{cases}$

Ou se donne les débits : $\overline{Q_L}$ et $\overline{Q_G}$
 Ou cherche : $\overline{R_G}$

Définition des débits:

$\boxed{1/A} \quad \overline{Q_G} = \langle \alpha \overline{w_G^x} \rangle = \overline{R_G} \overline{w_G^x}$

$\boxed{1/A} \quad \overline{Q_L} = \langle (1-\alpha) \overline{w_L^x} \rangle = (1-\overline{R_G}) \overline{w_L^x}$

Ou calcule $\overline{R_G}$

$$\overline{R_G} = \frac{\overline{Q_G} \overline{w_L^x}}{\overline{Q_L} \overline{w_G^x} + \overline{Q_G} \overline{w_L^x}} = \frac{\beta}{1 + \frac{(1-\overline{R_G})(\overline{w_G^x} - \overline{w_L^x})}{J}}$$

Rappel $J = \frac{\overline{Q}}{A} = \frac{\overline{Q_L} + \overline{Q_G}}{A}$

Exemple : Ecoulement à bulles w_{00} vit. bulle

$$\overline{w_G^x} - \overline{w_L^x} \approx w_{00} (1 - \overline{R_G})$$

- Corrélation pour w_{00} ($D, \sigma, \rho_L, \rho_G, \mu_L$ etc...)
 Clift par exemple
- Diagramme de Wallis

DIAGRAMME de WALLIS

• Définitions

$$j_k \triangleq \alpha_k \overline{w_k^x} = \overline{x_k w_k} \quad j = j_L + j_G$$

vitesse de dérive (vitesse relative / c du vol)

$$v_{kj} \triangleq \overline{w_k^x} - j$$

flux de dérive (flux de phase k / c du volume)

$$j_{GL} \triangleq \alpha_G (\overline{w_G^x} - j)$$

• Hypothèse 1D

$$J_{GL} = \star j_{GL} \star = \overline{R_G} (\overline{w_G^x} - J) = (1 - \overline{R_G}) J_G - \overline{R_G} J_L \quad (1)$$

$$J = J_G + J_L = \overline{R_G} \overline{w_G^x} + (1 - \overline{R_G}) \overline{w_L^x}$$

$$\Rightarrow J_{GL} = \overline{R_G} (1 - \overline{R_G}) (\overline{w_G^x} - \overline{w_L^x})$$

• Corrélation pour $\overline{w_G^x} - \overline{w_L^x}$ en éc à bulles

$$J_{GL} = w_{00} R_G (1 - R_G)^2 \quad (2)$$

• Diagramme de Wallis

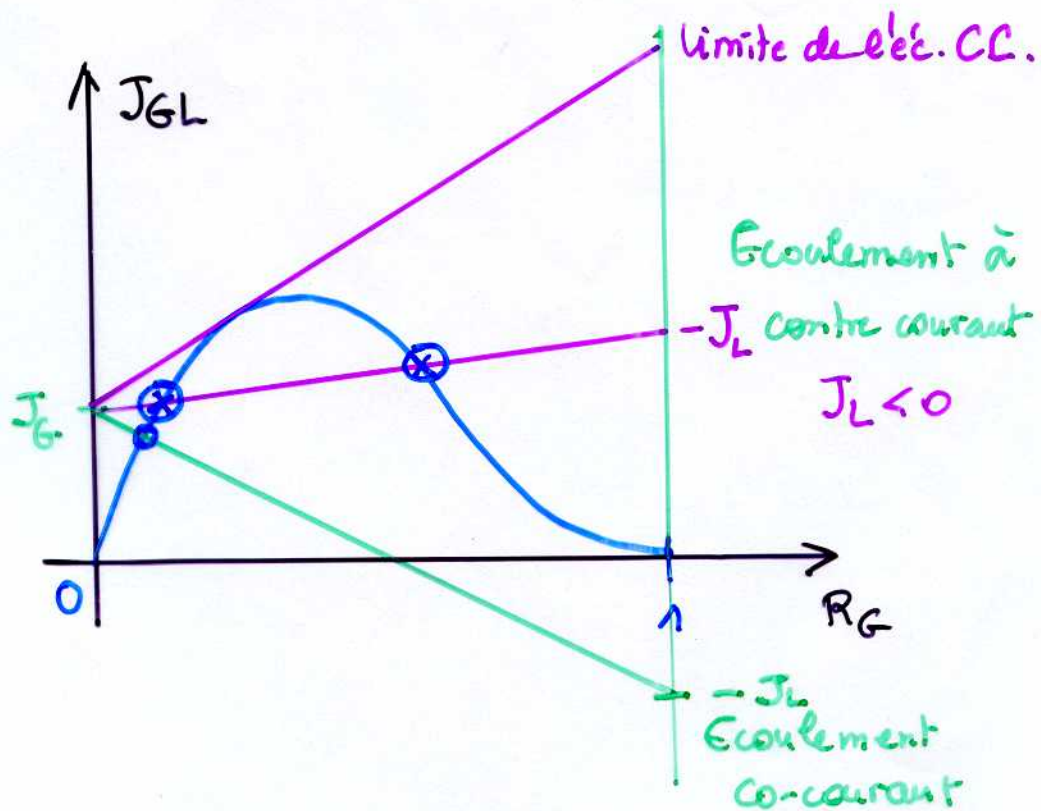
$$J_{GL} = f(R_G)$$

① droite

② courbe

$R_G = 0 ; J_G$

$R_G = 1 ; -J_L$



- Ecoulement co-courant ascendant.
 - 1 seul régime. ●
- Ecoulement à contre courant
 - 2 régimes peuvent coexister ⊗

MODELE de Zuber Findlay

• 2D-2V

• le plus général : on se donne $\overline{Q}_G$ et $\overline{Q}_L$
ou recherche $\overline{R}_G$

On part de la vitesse de dérive locale

$$\overline{w}_{Gj}^x \triangleq \overline{w}_G^x - j = (1 - \alpha_G)(\overline{w}_G^x - \overline{w}_L^x)$$

On moyenne sur la section le flux de dérive

$$\langle \alpha_G \overline{w}_{Gj}^x \rangle = \langle \alpha_G \overline{w}_G^x \rangle - \langle \alpha_G j \rangle$$

$$\langle \alpha_G \rangle \tilde{w}_{Gj} = J_G - C_0 \langle \alpha_G \rangle \langle j \rangle$$

$$R_G = \frac{J_G}{C_0 J + \tilde{w}_{Gj}}$$

$$\tilde{w}_{Gj} \triangleq \frac{\langle \alpha \overline{w}_{Gj}^x \rangle}{\langle \alpha_G \rangle}$$

$$C_0 = \frac{\langle \alpha_G j \rangle}{\langle \alpha_G \rangle \langle j \rangle}$$

• dégenère sur les autres modèles

• Expériences $\rightarrow$ correlations par régime

$$\frac{J_G}{R_G} = FL(J) \Rightarrow C_0 \text{ et } \tilde{w}_{Gj}$$

Le modèle de Zuber et Findlay

Ce modèle conduit à l'expression suivante du taux de vide :

$$R_G = \frac{\beta}{C_0 + \tilde{v}_{gj}/J} \quad ; ;$$

où β est le titre volumique et $J \hat{=} J_G + J_L$ la vitesse débitante du mélange ; C_0 paramètre de distribution et la vitesse de dérive pondérée $\tilde{v}_{gj}$ dépendent du type d'écoulement.

Les expressions suivantes peuvent être recommandées pour les conduites verticales :

$$C_0 = \left(1,2 - 0,2 \sqrt{\frac{\rho_G}{\rho_L}} \right) (1 - e^{-18R_G}) \quad \text{wall peaking.}$$

Écoulements à bulles :

$$\tilde{v}_{gj} = (C_0 - 1)J + 1,4 \left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2} \right]^{0,25} (1 - R_G)^{1,75}$$

Écoulements à poches :

$$\tilde{v}_{gj} = (C_0 - 1)J + 0,35 \left[\frac{g(\rho_L - \rho_G)D}{\rho_L} \right]^{0,5}$$

Écoulements agités :

$$\tilde{v}_{gj} = (C_0 - 1)J + 1,4 \left[\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2} \right]^{0,25}$$

Écoulements annulaires :

$$\tilde{v}_{gj} = \frac{1 - R_G}{R_G + \left[\frac{1 + 75(1 - R_G)}{\sqrt{R_G}} \frac{\rho_G}{\rho_L} \right]^{0,5}} \left[J + \sqrt{\frac{(\rho_L - \rho_G)gD(1 - R_G)}{0,015\rho_L}} \right]$$