# My solution for:
# Void fraction calculations from experimental data

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## 1 Data processing procedure

### 1.1 Mass rates measurements

The mass rate of gas is expressed in a volumetric unit: NL/min. However it is a true mass rate in kg/s. The conversion is done by using the value of the gas density $\rho_{G0}$ in the following reference state,

- Pressure, $p_0$: 1 atm, ≡1.01325 bar
- Temperature, $T_0$: 0°C≡ 273.15 K.
- for air $\rho_{G0} = 1.2928$ kg/m$^3$

as a result, the mass rate is given by,

$$M_G[\text{kg/s}] = M_G[\text{Nl/min}]\rho_{G0}\frac{10^{-3}}{60}. \tag{1}$$

The density of air is assumed to follow the perfect gas relation,

$$\rho_G = \rho_{G0}\frac{pT_0}{p_0T}, \tag{2}$$

as a result, the volumetric flow rate of gas is given by,

$$Q_G[\text{m}^3/\text{s}] \triangleq \frac{M_G}{\rho_G} = M_G[\text{Nl/min}]\frac{\rho_{G0}}{\rho_G}\frac{10^{-3}}{60} = M_G[\text{Nl/min}]\frac{p_0T}{pT_0}\frac{10^{-3}}{60} \tag{3}$$

The evaluation of the volumetric flow rate requires the knowledge of pressure and temperature.

### 1.2 Superficial velocity

By definition, the superficial velocity is the ratio of the volumetric flow rate to the cross sectional area. The pipe diameter is 49 mm. The area of its cross section is given by,

$$A = \pi\frac{D^2}{4} = 18.857 \text{ cm}^2 \tag{4}$$

The liquid flow rate is given in l/min. The liquid superficial velocity is given by,

$$J_L \triangleq \frac{Q_L}{A} = \frac{Q_L[\text{l/min}]\,10^{-3}}{60\,18.857\,10^{-4}}. \tag{5}$$

If we express the superficial velocity in cm/s we get finally,

$$J_L[\text{cm/s}] = \frac{Q_L[\text{l/min}]\,10^{-1}}{60\,18.857\,10^{-4}} = 0.8838\,Q_L[\text{l/min}] \tag{6}$$

The same calculation for the gas provides,

$$J_G[\text{cm/s}] = \frac{M_G[\text{Nl/min}]\,10^{-1}}{60\,18.857\,10^{-4}}\frac{p_0T}{pT_0} = 0.8838\,M_G[\text{Nl/min}]\frac{p_0T}{pT_0} \tag{7}$$

Calculating the superficial velocity **requires calculating the pressure** at the location of the NMR measurement, see figure 1.

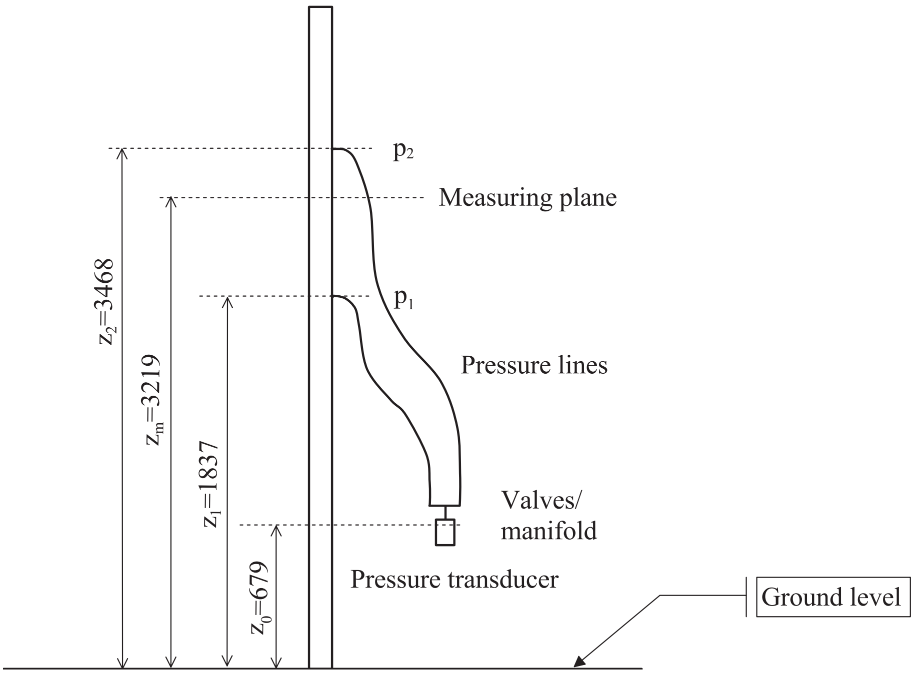



**Figure 1:** Schematic of the test section showing the location of the pressure taps, the pressure transducer and measuring section.

### 1.3 Absolute pressure calculation

We will assume that the flow is fully developed. The pressure distribution is therefore linear with the elevation. By considering that $z$ is oriented against the gravity direction, and that pressure always decreases with elevation as well as in the flow direction, $p_{s2}$ must be less than $p_{s1}$ and,

$$p(z) = \frac{p_{s1}(z_2 - z) + p_{s2}(z - z1)}{z_2 - z_1} \qquad (8)$$

where $p_{sk}$ is the static pressure at location $k$. The static pressure is related to the measured pressure by,

$$p_{sk} = p_k - \rho_L g(z_k - z_0) \qquad (9)$$

where the last term is the mere contribution of the the liquid weight in the piping connecting the pressure tap to the pressure transducer. By introducing this last relation into (8), one has,

$$p(z_m) = 0.1527\, p_1 + 0.8473\, p_2 - 0.2487 \text{ [bar]} \qquad (10)$$

where the elevations shown in figure 1 have been used.

### 1.4 Void fraction calculation: RG3

A momentum balance between section 1 and 2, projected along the vertical upward direction reads,

$$A(p_{s1} - p_{s2}) - A\rho_m g(z_2 - z_1) - \mathcal{P}\tau_W = 0 \qquad (11)$$

where $\tau_W$ is the friction stress at the wall and $\mathcal{P} = \pi D$ is the wetted perimeter of the wall. By introducing the frictional pressure drop, $\Delta p_F = 4/D\tau_W$, one gets,

$$\rho_m = \frac{p_{s1} - p_{s2} - \Delta p_F}{gH} \approx (1 - R_{G2})\rho_L \tag{12}$$

The frictional pressure drop is estimated via the homogenous model and the Blasius single-phase pressure drop model,

$$\begin{aligned} &\mathrm{Re} = \frac{GD}{\mu}, \quad C_F = 0,079\ \mathrm{Re}^{-0,25}, \quad \tau_W = \frac{1}{2}C_F\rho_H v_H^2, \\ &\Delta p_F = \frac{4H\tau_W}{D}, \quad \rho_H = (1-\beta)\rho_L + \beta\rho_G, \quad \rho_H v_H = G. \end{aligned} \tag{13}$$

A liquid viscosity value $\mu = 0.001$ Pa/s can be assumed. The results of the data processing is shown in Table 1.

## 2 Void fraction models

For the analytic determination of the void fraction with the 4 models, we will assume a constant value of the liquid superficial velocity, $J_L = 35.6$ cm/s, and a variable gas superficial velocity, $J_G$ ranging from 0 to 20 cm/s. When necessary a constant value of the pressure in the measuring section will be assumed, $p \approx 1.17$ bar.

### 2.1 The homogeneous model

According to the homogeneous model, the void fraction is equal to the volumetric gas quality, $\beta$,

$$R_G = \beta = \frac{J_G}{J_G + J_L} \tag{14}$$

### 2.2 The Bankoff model

According to the Bankoff model, the void fraction is given by,

$$R_G = K\beta, \quad K = 0.71 + 0.00145p\ [\mathrm{bar}] \tag{15}$$

where the value of $K \approx 0,71$ in our conditions.

### 2.3 The Wallis model

In this two-velocitiy 1D-model, the mean velocity difference between the phases, at vanishing value of the void fraction, is given by,

$$u_\infty = 1.53\left(\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right)^{0.25} \approx 24.95\ \mathrm{cm/s} \tag{16}$$

Note that the evaluation of the void fraction with this model requires iterating on the void fraction values since the velocity difference depends on the void fraction and that the following equation is implicit in the void fraction values,

$$R_G = \frac{\beta}{1 + \dfrac{(1-R_G)(w_G - w_L)}{J}}, \quad w_G - w_L = u_\infty(1 - R_G)^{0.5} \tag{17}$$

A numerical solution of (17) is therefore necessary. Is is suggested solve iteratively for $R_G$ and to start with the value given by the Bankoff model, $R_G \approx 0.7\beta$.

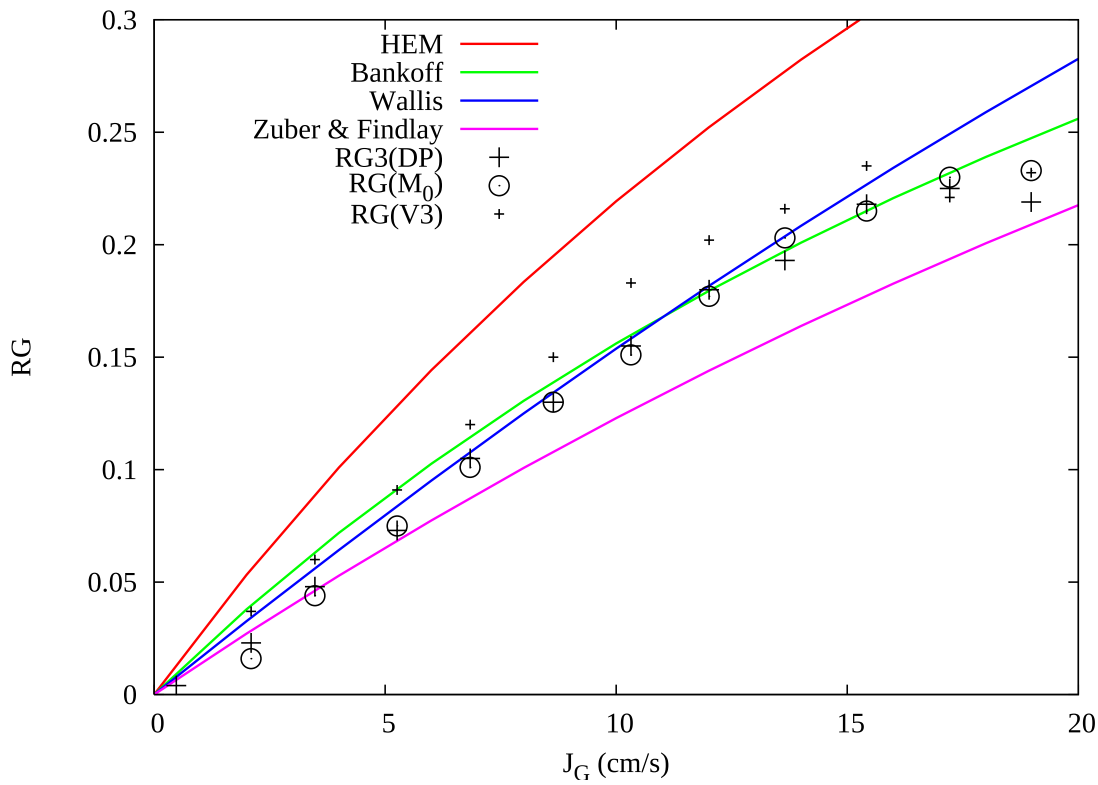



**Figure 2:** Void fraction predictions with the four models against the data of table 1. Code and script : `05PC01`, file : `05PC-alpha`.

### 2.4 The Zuber & Findlay model

According to this model, closures are flow regime dependent. Since the observation shows that flow regimes are of bubbly and short slug type at the exit of the test section for the highest values of the gas rate, we will assume bubbly flow all throughout for simplicity. Therefore, the following applies,

$$C_0 = \left(1,2 - 0,2\sqrt{\frac{\rho_G}{\rho_L}}\right) \approx 1.193 \tag{18}$$

$$\begin{aligned}\widetilde{w}_{GJ} &= (C_0 - 1)J + 1,4\left(\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right)^{1/4}(1 - R_G)^{7/4} \\ &\approx 0.193J + 22.83(1 - R_G)^{7/4}\ [\text{cm/s}]\end{aligned} \tag{19}$$

The void fraction calculation also requires iteration. The procedure detailed in the previous subsection may be applied,

$$R_G = \frac{\beta}{C_0 + \dfrac{\widetilde{w}_{GJ}}{J}} \tag{20}$$

## 3 Data analysis and conclusions

From figure 2, it can be seen that both determinations of the void fraction are consistent ($\Delta P$ method and NMR). However, the consistency check provided by the mean velocity measured by NMR and the known value of the liquid superficial velocity is away from the two other estimates. The table 1 shows that combining the void fraction and mean velocity determined by NMR provides a consistent value of the superficial velocity; error compensations are occurring here obviously.

The data shows a change of trends at a void fraction values larger than 15 %. Observation reveals bubble clustering beyond this value with large scale wake behind them. Further downstream Taylor bubbles appear and can be considered as the onset of the slug flow transition. Table 1 also shows a rather rapid increase of velocity fluctuations (vp3).

Obviously the homogenous model over-predicts by 30% the void fraction as expected. The Zuber and Findlay model seems to under-predict the data, though it could be considered as a more sophisticated model. It is however clear that both the Bankoff and Wallis model give reasonable results. This is a typical situation in two-phase flow models. these models are based on two radically distinct views of the phenomenon and have been fitted on some data.

- The same data is reproduced by assuming that the deviation from the homogeneous model is a result (i) of non-flat velocity profiles in the Bankoff's model; (2) of a velocity difference between the phases in the Wallis' model.
- The Zuber & Findlay model is supposed to account for both effects and seems to be less accurate that the two former simpler models.

To give a final judgment one should know:

- The validity range of each model, not indicated in the lecture nor in textbooks. By validity range it is meant at least the type of flow conditions used to derive the closure relations. In order to get this information one must refer to the original work.
- The quality of the design of the experiment. Is the flow fully developed? Is it properly conditioned? There is no guarantee that the test section can be fairly compared with those which provided the models data (injection conditions).
- The uncertainty in the data. It is not known yet for NMR data and for the pressure drop methods, it is very poor with our selected technology. A rather optimistic estimated gives $\pm 1,5$ % of void fraction absolute. If this type of accuracy is to be considered with NMR, the data cannot discriminate the models.

## 4 Some comments on data fitting

Figure 2 show some reasonable agreement of the data with the Wallis model. It could be considered to fit the model to our particular conditions. The fit has been performed in the coordinates of the Wallis diagram shown in figure 3. The fitted value of $u_\infty = 28$ cm/s is slightly larger than that given by the model ($\approx 25$ cm/s). The order of magnitude is correct, though to get it, only the points with void fraction $0.1 < R_{G2} < 0.2$ have been considered. Points with the smaller void fraction values were considered inaccurate.

The Zuber & Findlay model seems less accurate when compared with the data. The figure 4 shows clearly that drawing a line in this diagram with points at only one value of the liquid superficial velocity is a challenging guess. To get the constant values

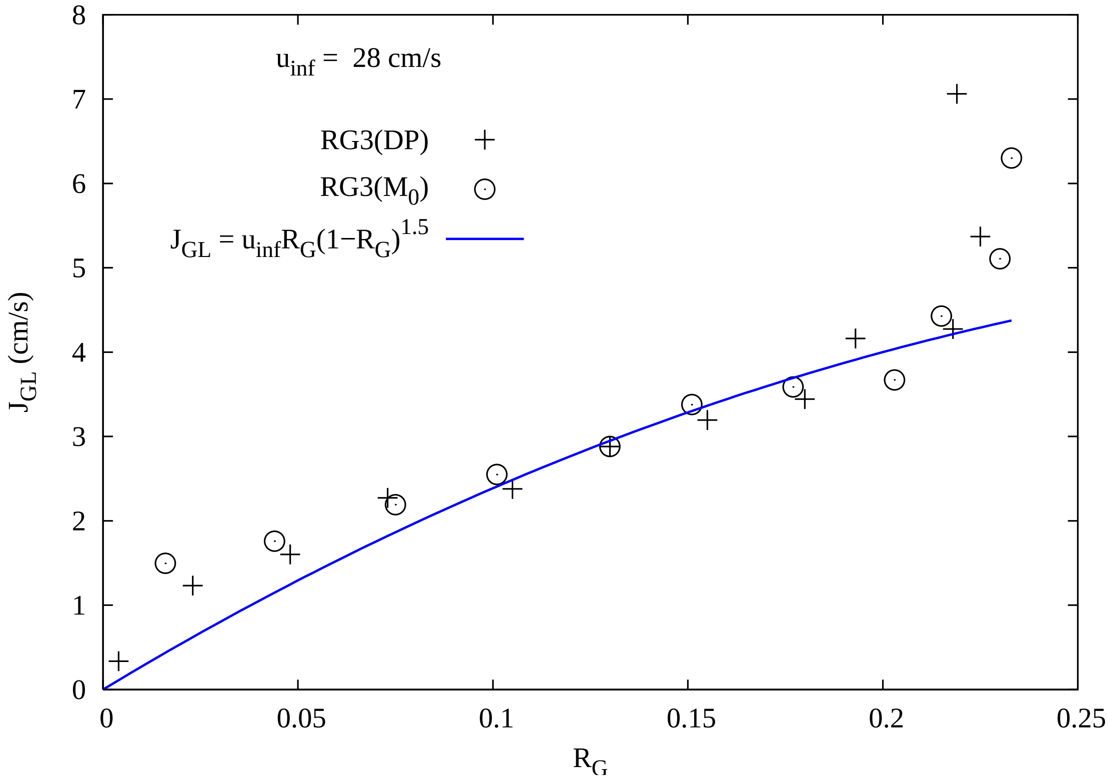



**Figure 3:** Data of table 1 shown in the coordinates of the Wallis diagram: drift-flux vs void fraction. The solid line represents the data fit with equation shown in the figure caption.

shown in the figure caption, only points with void fraction values $R_G > 0.1$ have been considered. The values of the correlation coefficient $C_0 \approx 1.04$ and of the weight-averaged drift velocity, $\widetilde{w}_{Gj} \approx 20$ cm/s agree with the model prediction. If there were no data selection, the fitted coefficients may take arbitrary values. Again, the data set contains data with only one value of the liquid superficial, and it is not sufficiently wide to provide an accurate fit of the data.

The figure 5 shows the data fitting procedure in a different coordinate set: $R_G$ vs $J_G$. In figure 5(a) the points considered in figure 5(b) are limited to the smaller void fraction values ($R_G < 0.2$). Clearly the fitted values, though they provide a better fit to the data, are rather inconsistent with the model perditions. The following remarks apply:

- Data fitting results may depend strongly on the coordinate selected coordinate system.
- Uncertainty in the fitted parameter is a better measure of the fit quality than the mean square residue of the fit. Some theoretical background for this is provided for example by Press *et al.* (1992, Sec. 15.6).

## References

Press, W. H., Teukolski, S. A., Vetterling, W. T., & Flannery, B. P. 1992. *Numerical recipes in fortran, the art of scientific computing*. Second edn. Cambridge Unviersity Press.

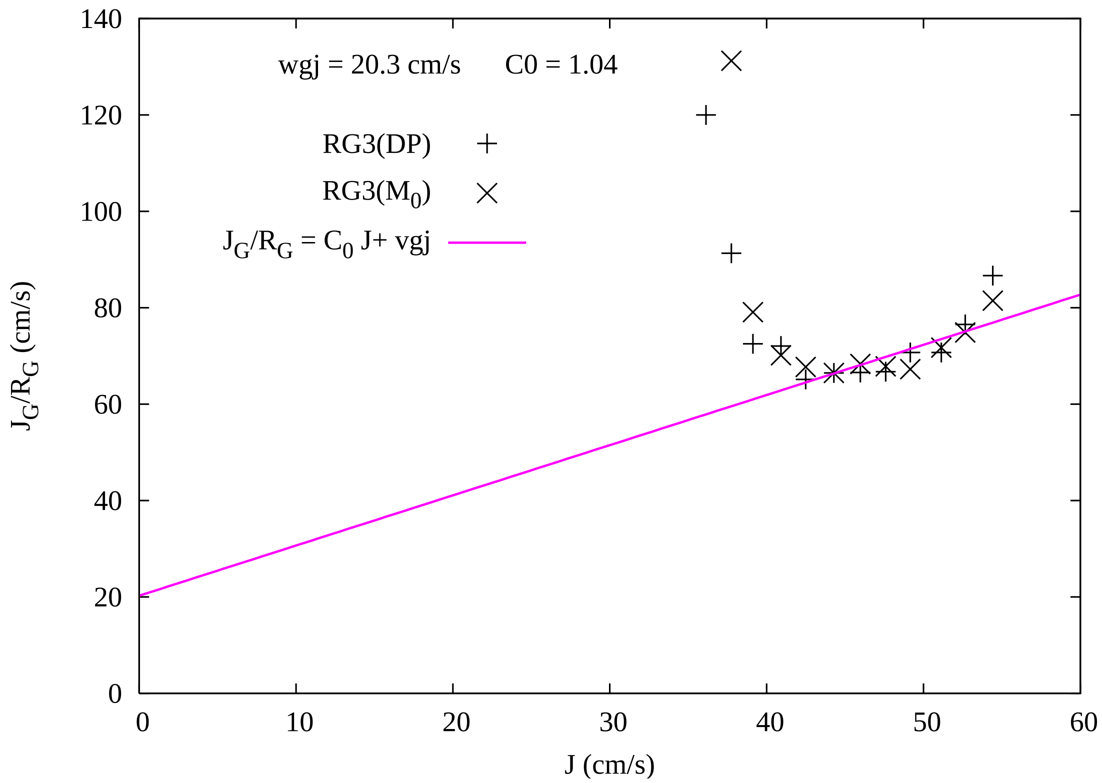
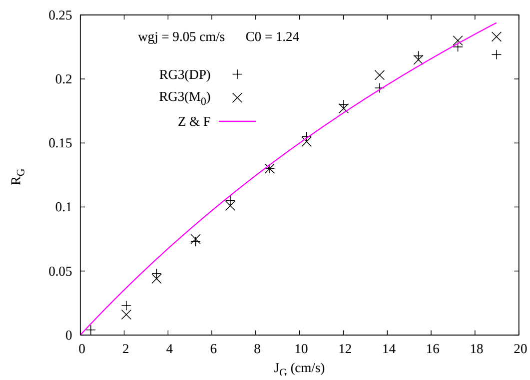
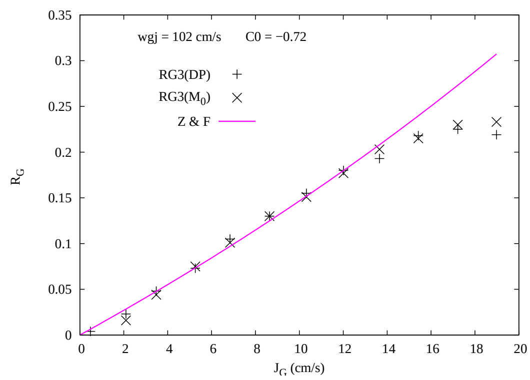



**Figure 4:** Data of table 1 shown in the coordinates of the Zuber &Findlay diagram: $J_G/R_G$ vs the total superficial velocity ($J$). The solid line represents the data fit with equation shown in the figure caption.




(a) $0.1 < R_G < 0.3$




(b) $0 < R_G < 0.2$

**Figure 5:** Fit of the Zuber & Findlay model on the data of table 1. (a) Considered points are identical to those of figure 4. (b) Points with void fraction $R_G < .20$ are only considered for fitting the constants.

| Run | JL | TL | P | JG | RG3 | beta | v3 | alp. | alp. | JLRMN | vp3 | D |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | cm/s | °C | bar | cm/s | % | % | cm/s | v3% | M0% | cm/s | cm/s | cm2/s |
| J668 | 35.65 | 20.5 | 1.197 | .48 | .4 | 1.3 | 35.56 | -.2 | -.3 | 35.65 | 8.57 | .1112 |
| 0669 | 35.65 | 20.3 | 1.192 | 2.10 | 2.3 | 5.6 | 37.00 | 3.7 | 1.6 | 36.40 | 9.25 | .1653 |
| 0670 | 35.65 | 20.3 | 1.187 | 3.48 | 4.8 | 8.9 | 37.91 | 6.0 | 4.4 | 36.22 | 10.58 | .2103 |
| 0671 | 35.65 | 20.3 | 1.181 | 5.26 | 7.3 | 12.8 | 39.22 | 9.1 | 7.5 | 36.28 | 11.65 | .2875 |
| 0672 | 35.65 | 20.3 | 1.175 | 6.84 | 10.5 | 16.1 | 40.49 | 12.0 | 10.1 | 36.39 | 12.19 | .3078 |
| 0674 | 35.65 | 20.3 | 1.169 | 8.64 | 13.0 | 19.5 | 41.92 | 15.0 | 13.0 | 36.48 | 13.26 | .3825 |
| 0675 | 35.65 | 20.3 | 1.166 | 10.32 | 15.5 | 22.4 | 43.61 | 18.3 | 15.1 | 37.05 | 14.16 | .4618 |
| 0676 | 35.58 | 20.3 | 1.161 | 12.01 | 18.0 | 25.2 | 44.57 | 20.2 | 17.7 | 36.68 | 15.36 | .5502 |
| 0677 | 35.51 | 20.8 | 1.158 | 13.65 | 19.3 | 27.8 | 45.32 | 21.6 | 20.3 | 36.12 | 16.32 | .5459 |
| 0678 | 35.71 | 20.8 | 1.156 | 15.42 | 21.8 | 30.2 | 46.66 | 23.5 | 21.5 | 36.64 | 21.53 | .8990 |
| 0679 | 35.44 | 20.8 | 1.153 | 17.22 | 22.5 | 32.7 | 45.52 | 22.1 | 23.0 | 35.06 | 30.14 | 1.8860 |
| 0680 | 35.44 | 20.8 | 1.150 | 18.98 | 21.9 | 34.9 | 46.15 | 23.2 | 23.3 | 35.41 | 33.68 | 1.9660 |

Input file :`0668-0680.in` and USE MG50 option, $M_\infty = 18875$. $T_1 = 2.576$ s

**Table 1:** Data processing of the series of runs `0668-0689`. Use MG50 data.

| $J_L$ | $J_G$ | HEM | Bankoff | Wallis | Zuber & F. |
|---|---|---|---|---|---|
| cm/s | cm/s | - | - | - | - |
| 35.60 | .00 | .0000 | .0000 | .0000 | .0000 |
| 35.60 | 2.00 | .0532 | .0379 | .0326 | .0271 |
| 35.60 | 4.00 | .1010 | .0719 | .0643 | .0529 |
| 35.60 | 6.00 | .1442 | .1026 | .0951 | .0774 |
| 35.60 | 8.00 | .1835 | .1306 | .1250 | .1008 |
| 35.60 | 10.00 | .2193 | .1561 | .1538 | .1229 |
| 35.60 | 12.00 | .2521 | .1794 | .1816 | .1439 |
| 35.60 | 14.00 | .2823 | .2009 | .2084 | .1639 |
| 35.60 | 16.00 | .3101 | .2207 | .2342 | .1827 |
| 35.60 | 18.00 | .3358 | .2390 | .2589 | .2006 |
| 35.60 | 20.00 | .3597 | .2560 | .2826 | .2176 |

**Table 2:** Numerical values of the predicted void fractions with the the four models.