A SHORT INTRODUCTION TO TWO-PHASE FLOWS

Industrial occurrence and flow regimes

Hervé Lemonnier
DM2S/STFM/LIEFT, CEA/Grenoble, 38054 Grenoble Cedex 9
Tél. 04 38 78 45 40
herve.lemonnier@cea.fr, *herve.lemonnier.sci.free.fr/TPF/TPF.htm*

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CLASSES CONTENTS (1/2)

- Introduction: CEA/Grenoble, scientific information
- Two-phase flow systems in industry and nature
- Flow regime
- Measuring techniques, composition (α)
- Simple models for void fraction prediction

CLASSES CONTENTS (2/2)

- Balance equations
- 1D models, pipe flow
- Pressure drop and friction
- Heat transfer mechanisms in boiling
- Condensation of pure vapor
- Critical flow phenomenon

Recommended textbook: Delhaye (2008)

THERMAL-HYDRAULICS

- Study of simultaneous flow and heat transfer, in French, *thermohydraulique*
- Phase: state of matter characterized by definite thermodynamic properties
- Two-phase: mixture of two phases (*diphasique*)
- Examples: air and water, oil and water (connate), water and steam, oil and natural gas (multiphase), *polyphasique*).

CEA/GRENOBLE RESEARCH CENTER

- CEA: Commissariat à l'Energie Atomique (15000 p)
- CEA/Grenoble: originates in 1956, founded by Louis Néel (4000p/2300 CEA)
- Heat transfer laboratories founded by Henri Mondin
- Nuclear energy directorate (5000 p, DEN)
- Department of nuclear technology (400 p, Cadarache, Grenoble, DTN)
- Department of reactors studies (400 p, Cadarache, Grenoble, DER)
- Labs of simulation in thermal-hydraulics (SSTH)
- Labs of experimental studies in thermal-hydraulics (SE2T)
- Thesis advising capabilities and referenced research groups for several Masters.

LABS OF SIMULATION AND EXPERIMENTS IN THERMAL-HYDRAULICS

- Codes (*logiciels*) for safety studies, CATHARE.
- 3D Codes for two-phase boiling flows (Neptune).
- LES of single-phase flow and heat transfer (TRIO-U).
- Dedicated studies : safety and optimization of NR of various generations II, III and IV, ship propulsion, cryogenic rocket engines.
- Analytic studies on boiling flows and critical heat flux (DEBORA)
- Thermal-hydraulic qualification of fuel bundles (OMEGA)
- Instrumentation development for single-phase and two-phase flows:

Can only be modeled a quantity which can be measured

Applications→models and codes→experimental validation→instrumentation.

SCIENTIFIC KNOWLEDGE AND INFORMATION

- How to solve a technical/scientic issue?
- Textbooks, books, journal papers: Library?.
- Scientific Societies: journals editing, conference organizations (proceedings, *actes*).
  - La Société française de l'énergie nucléaire
  - La Société hydrotechnique de France
  - La Société française de thermique
  - American nuclear society, thermal-hydraulics division (NURETH)
- Do you speak English? ...

TWO-PHASE SYSTEMS IN INDUSTRY AND NATURE (1/4)

- Nuclear engineering: sizing, safety, decontamination (cleaning up)
  - Loss of coolant accidents (LOCA-*APRP*).
  - Severe accidents w/o vessel retention.
  - Decontamination by using foam.
  - Nuclear waste reprocessing.
- Oil engineering, hot issue: two-phase production
  - Transport.
  - Pumping.
  - Metering.
  - Oil refining (Chem. Engng).
- Oil engineering : safety
  - Safety of installations.
  - LPG storage tanks and fire (BLEVE).

TWO-PHASE SYSTEMS IN INDUSTRY AND NATURE (2/4)

- Chemical engineering
  - Wastewater treatment (interfacial area and residence time).
  - Gas-liquid reactors (falling film, trickle bed, air-lift).
  - Mixing and separation.
  - Safety: homogeneous thermal runaway.
- Automotive industry
  - Diesel fuel atomization.
  - Combustion in diesel engines.
  - Cavitation damage : power steering, fuel nozzles.
- Heat exchangers
  - Condensers and evaporator/steam generator.
  - Boilers (critical heat flux, CHF), heaters.

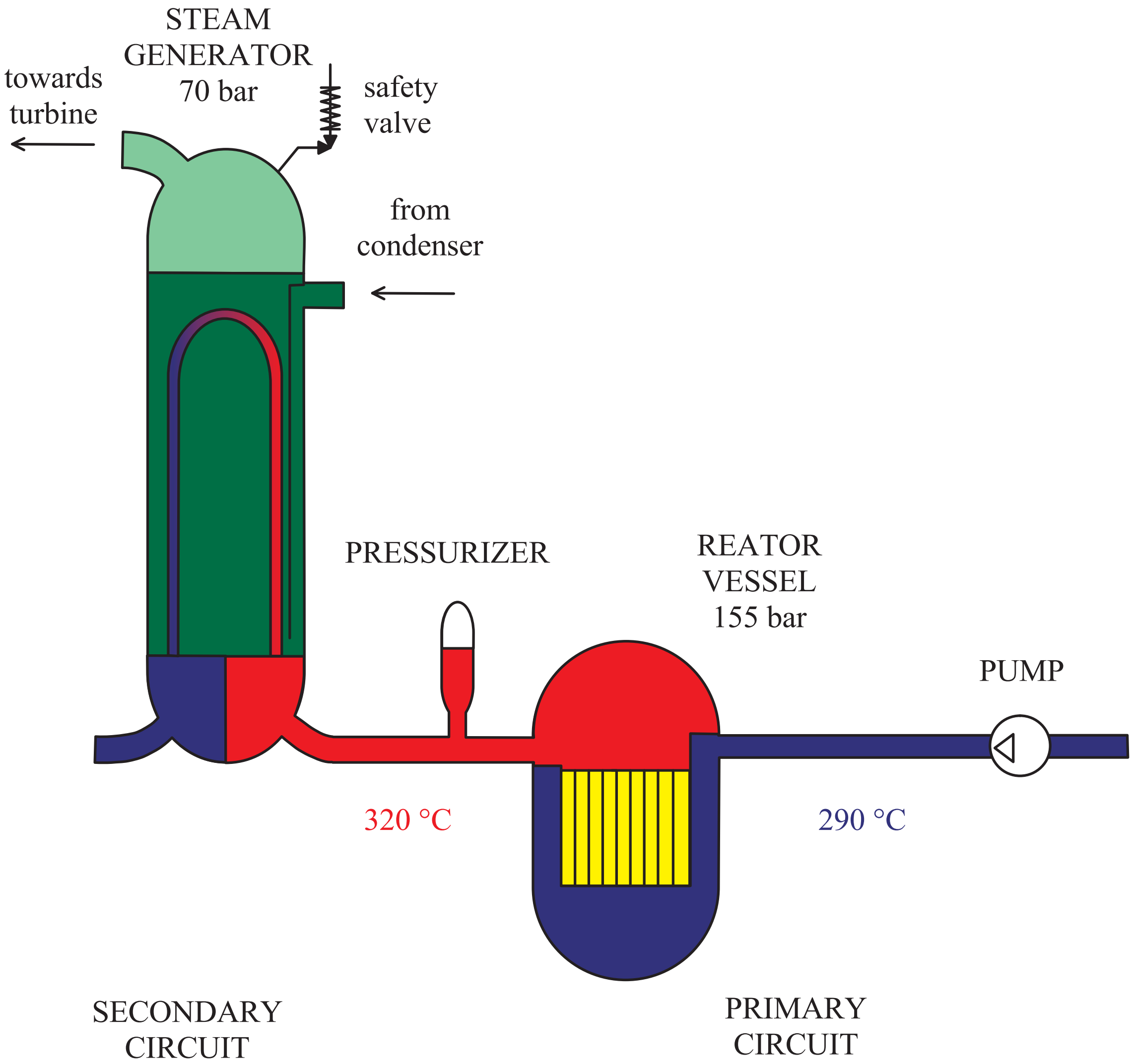
TWO-PHASE SYSTEMS IN INDUSTRY AND NATURE (3/4)

- Hydroelectricity and water distribution
  - Water resources management: transients of pipings.
  - Priming of siphons.
- Space industry
  - Cryogenic fuel storage (Vinci).
  - Thermal control of rocket engines (combustion chamber and nozzle).
  - Water hammer and pressure surges.
  - Cavitation in turbo-pumps. Instability (lateral loading) and damage.

TWO-PHASE SYSTEMS IN INDUSTRY AND NATURE (4/4)

- Meteorology
  - Storm formation, rain/hail, lightning.
  - Ocean and atmosphere exchanges, aerosols formation.
- Volcanology
  - Critical flow of lava in wells.
  - Steam explosion.
  - Nuées ardentes (Vesuvius, protection of Naples suburbs).
- Nivology
  - Avalanches.
  - Snow maturation (three-phase / 2-component).

NUCLEAR REACTORS, WATER COOLED



- Sizing :
  SG: heat transfer and pressure drop.
  SGTR: critical flow at the safety valve.
  FSI: mechanical loading and vibrations.
- Safety :
  LOCA, fuel cladding temperature, reference scenario
- Decontamination :
  vessel, SG, minimizings wastes: foam

NUCLEAR FUEL

- Fuel pellet.
- Rod ≈ 10 mm in diameter (first confinement barrier).
- Fuel assembly 17×17.
- Control rods.
- Length ≈ 4 meters.
- Reactor core ≈ 4 m in diameter.
- Heat transfer: forced convection (7 m/s.)
- Thermal power $3000 \div 5000$ MW.

STEAM GENERATOR

- Tube-type, separates primary and secondary circuits (second confining barrier).
- $\approx$ 5000 tubes, diameter 50 mm, height 10 m.
- Pressure: 155-70 bar.
- 3 or 4 SG and flow loops.
- Inverted U-Tubes.
- Secondary : two-phase flow.
- Issues : heat transfer and vibration damage.

THE N4 PWR: SOME FIGURES

- Primary side pressure: 155 bar, $T_{\text{sat}} \approx 355°\text{C}$. Thermal power: 4250 MW.
  - Mass flow rate: 4928,6 kg/s per SG (4)
  - Core inlet temperature: 292,2°C
  - Core outlet temperature: 329,6°C
- Secondary side, SG vapor pressure : 72,3 bar
  - Vapor temperature: 288 ° C
  - Feed water temperature: 229,5°C
  - Mass flow rate: 601,91 kg/s per SG (4)
- Assess the thermal balance of reactor and SG

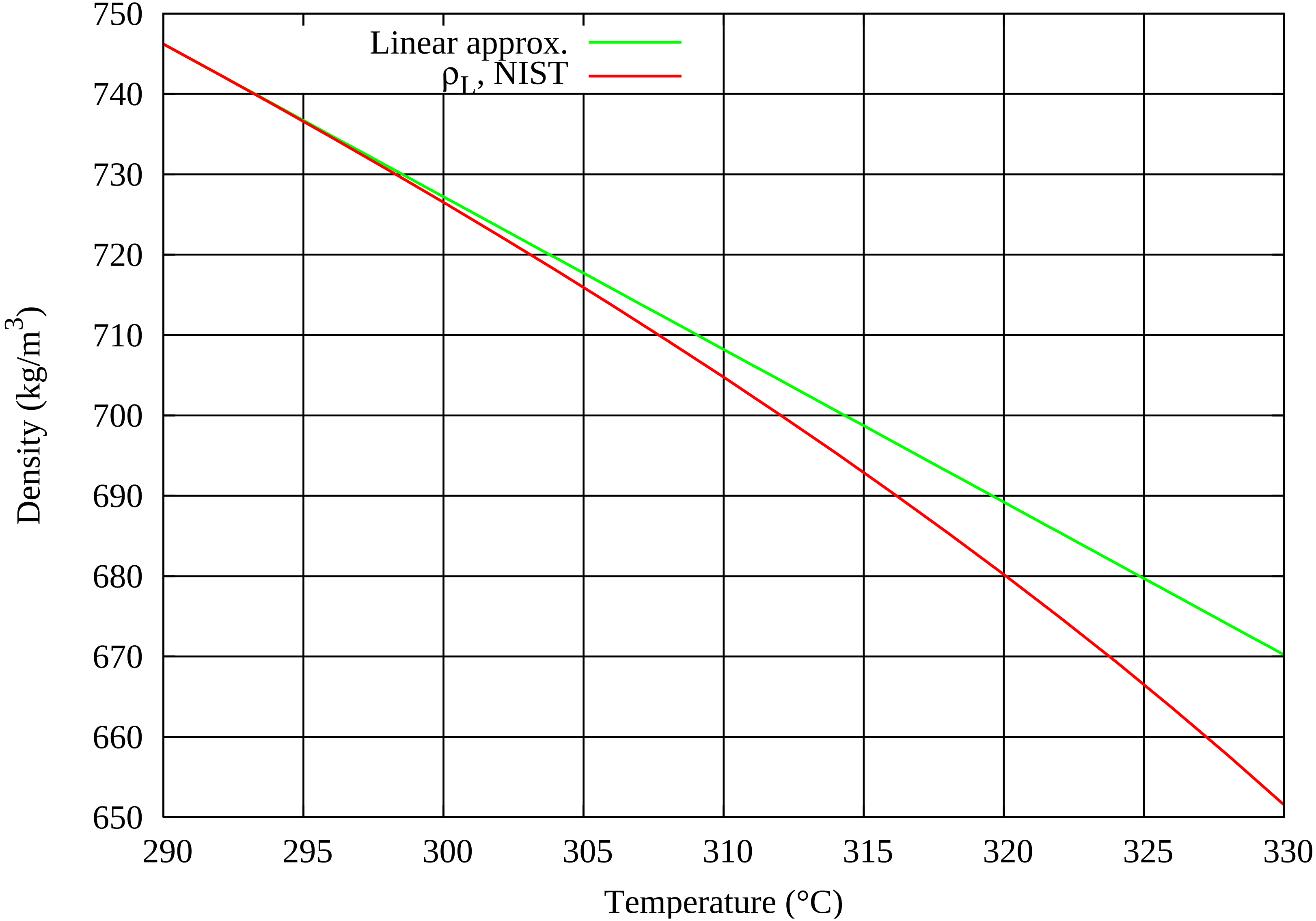
Source: National Institute of Standards and Technology (NIST)
(http://webbook.nist.gov/chemistry/fluid/)

SOME BAD NEWS...

The following (low pressure) statements are *rather* wrong:

- The mass balance reads, $Q_1 = Q_2$, since water is incompressible, at least weakly it is dilatable.
- The enthalpy is, $h = C_P T$.
- For a liquid, $C_P \approx C_V$, or $h \approx u$.
- Steam is a perfect gas.

WATER DENSITY AT 155 BAR



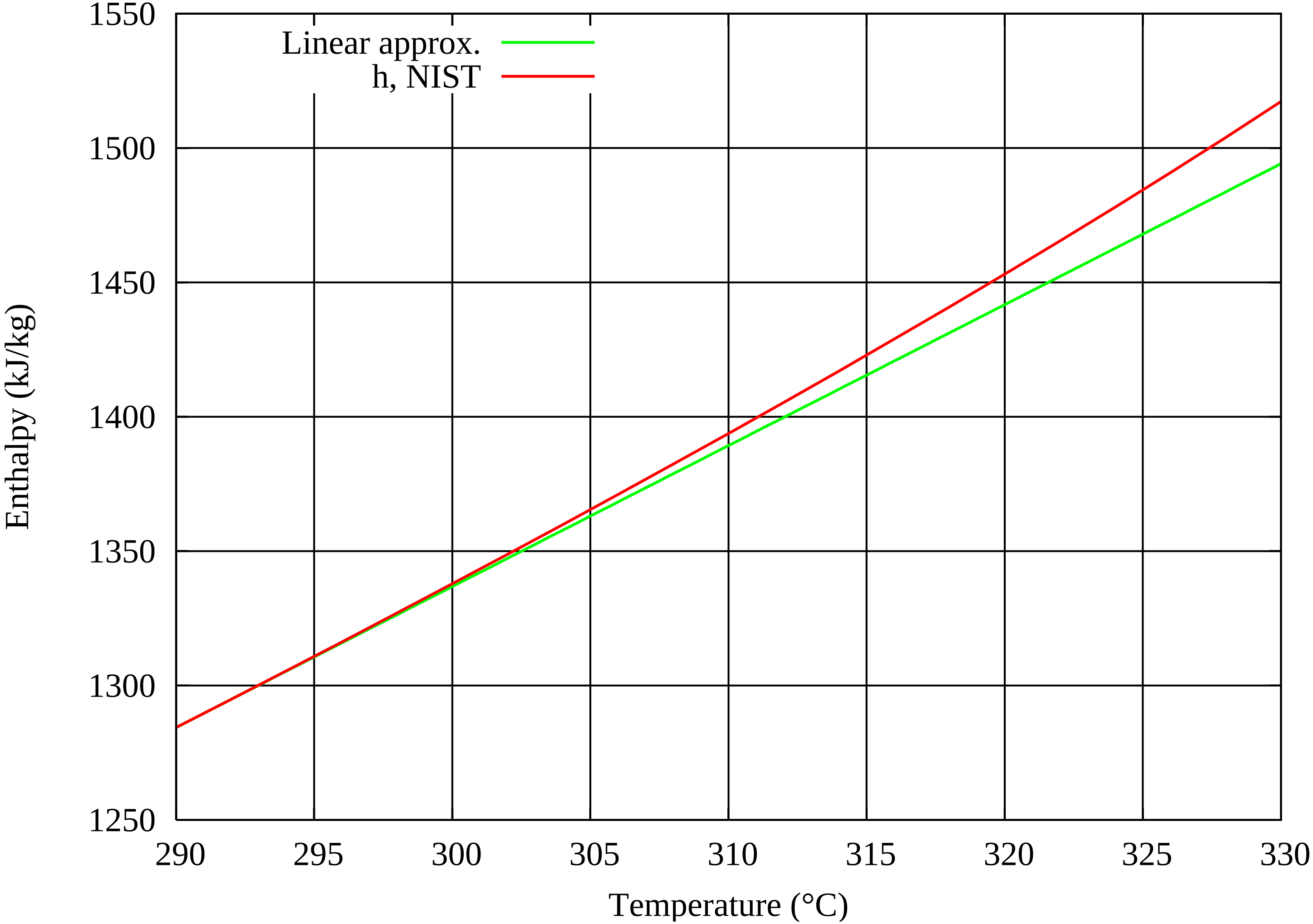
MASS BALANCE OF THE PRIMARY CIRCUIT

- Mass flow rate per loop (CL), $M_L \approx 5023$ kg/s
- Inlet density: $\rho_{L1}(292^{\mathrm{o}}\mathrm{C}, 155 \text{ bar}) = 742,41$ kg/s.
- Outlet density : $\rho_{L2}(330^{\mathrm{o}}\mathrm{C}, 155 \text{ bar}) = 651,55$ kg/s.

$$Q_1 = \frac{M_L}{\rho_1} = 6,77\mathrm{m}^3/\mathrm{s}, \quad Q_2 = \frac{M_L}{\rho_2} = 7,71\mathrm{m}^3/\mathrm{s}$$

- Volumetric flow rates differ by 13%.
- The volume of the primary circuit is 400 m^3...

WATER ENTHALPY AT 155 BAR



PRIMARY SIDE HEAT BALANCE

- Mass flow rate per loop (CL), $M_L \approx 5023$ kg/s.
- Inlet enthalpy: $h_{L1}(292^{o}\mathrm{C}, 155\mathrm{bar}) = 1295$ kJ/kg.
- Outlet enthalpy: $h_{L2}(330^{o}\mathrm{C}, 155\ \mathrm{bar}) = 1517$ kJ/kg.

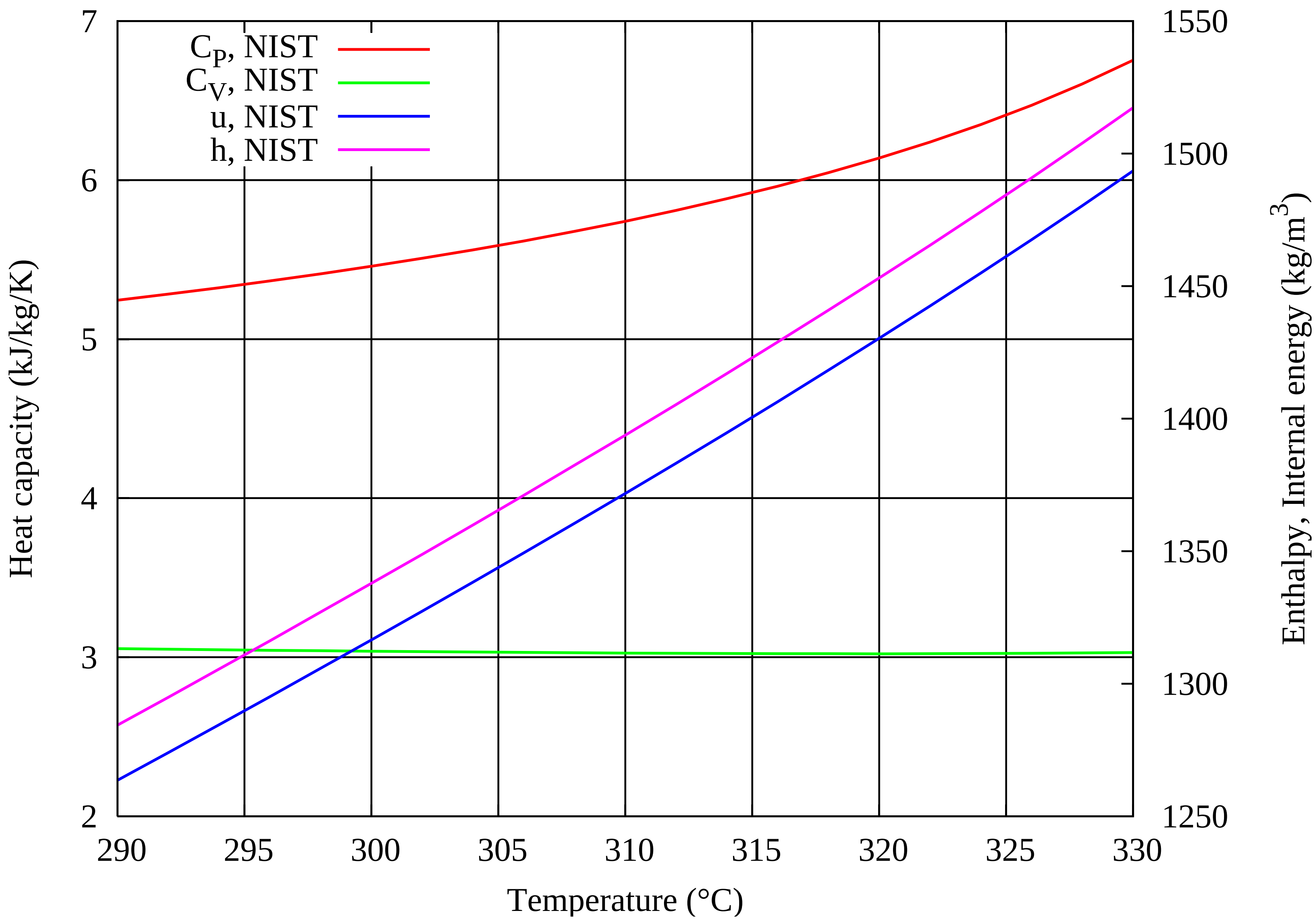
$$P = M_L \Delta h \approx 5023 \times 222\,10^3 = 1115\ \mathrm{MW}$$

- 4-loop reactor power: 4460 MW.
- Linear approximation: $h = C_P T$, $C_P(292^{o}\mathrm{C}, 155\mathrm{bar}) = 5,2827$ kJ/kg/K.

$$P = M_L C_P \Delta T \approx 5023 \times 201\,10^3 = 1008\ \mathrm{MW}$$

- Power differs by 10%.
- Temperature drift: 31°C/hour.

WATER ENTHALPY & INTERNAL ENERGY AT 155 BAR



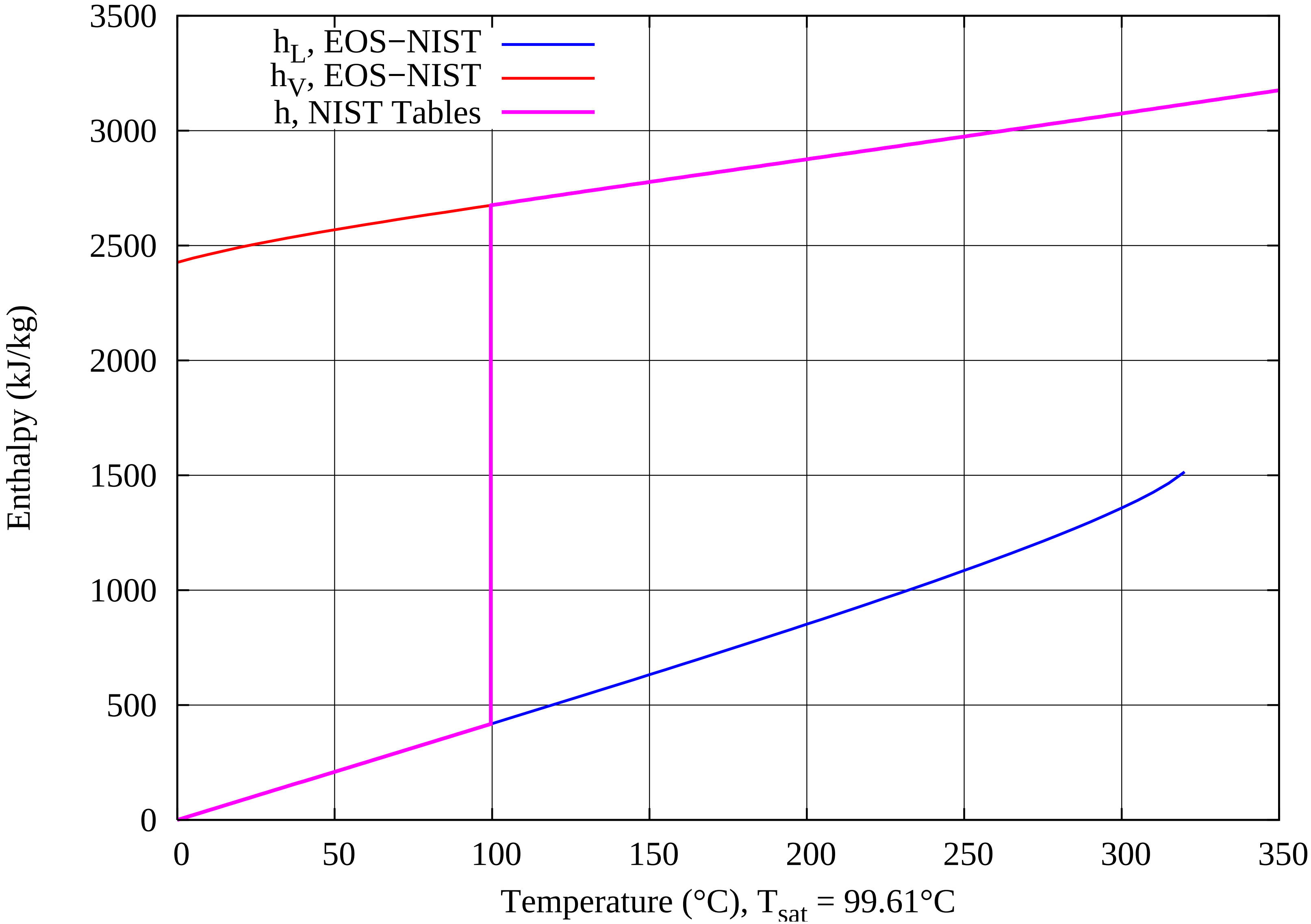
PHASE CHANGE & METASTABLE STATES

- Phase rule: thermodynamic equilibrium of steam and vapor, temperature and pressure are linked, saturation states

$$v = n + 2 - \varphi = 1, \quad p = p_{\text{sat}}(T), \text{ or } T = T_{\text{sat}}(p)$$

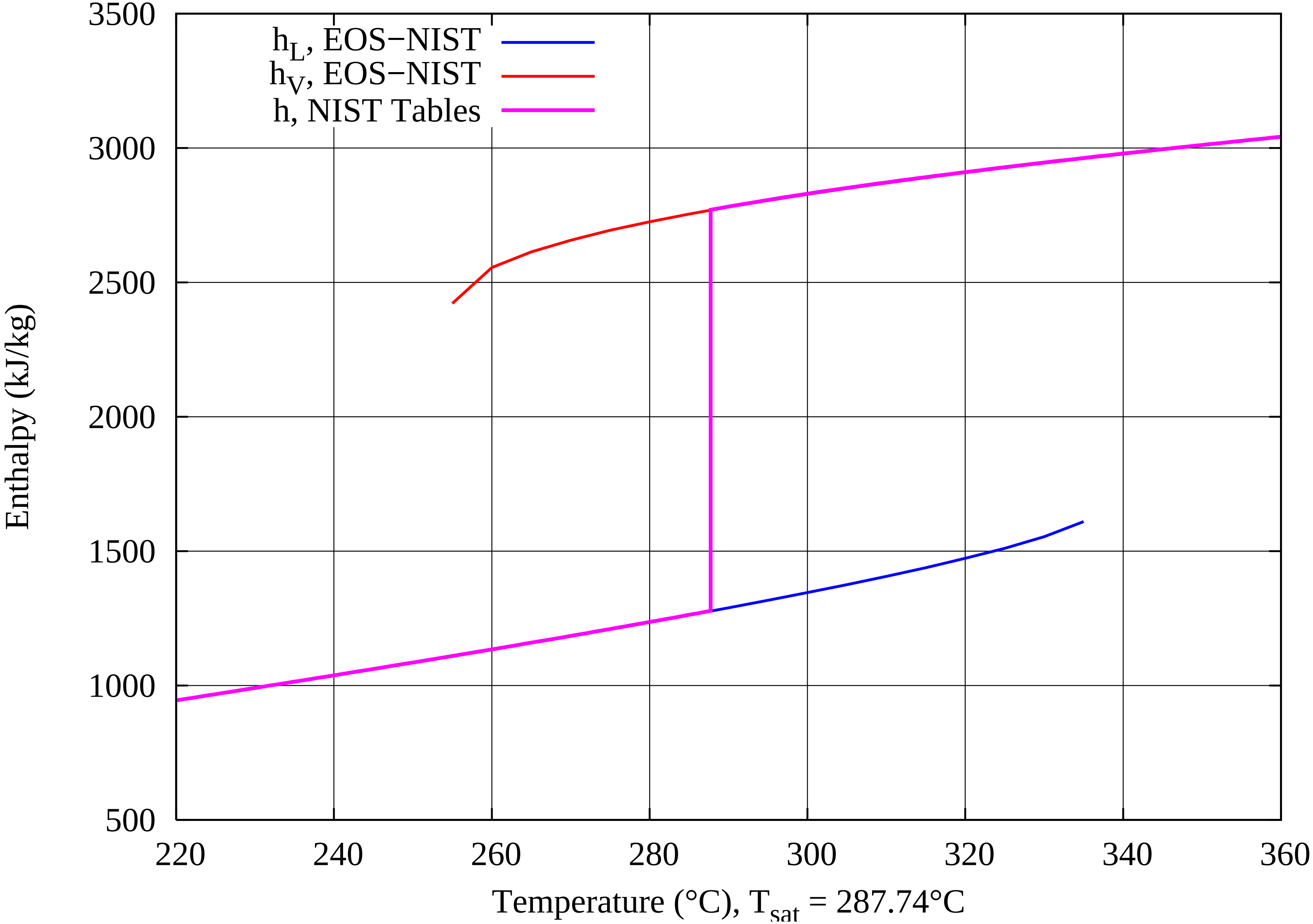
- Phase coexistence pressure, resp. temperature
- Liquid only is thermodynamically stable above $T_{\text{sat}}(p)$.
- Vapor only is thermodynamically stable below $T_{\text{sat}}(p)$.
- Water must boil and comply with the second principle of thermodynamics
- Within the metastability T range, the fluid can be either two-phase or single-phase.
- Metastable states usually not in tables, EOS is needed.

WATER ENTHALPY AT 1 BAR



Maximum liquid superheat, metastability limit >230 K

WATER ENTHALPY AT 72 BAR



SECONDARY SIDE HEAT BALANCE

- Mass flow rate per SG, $M_L \approx 602$ kg/s
- Inlet enthalpy: $h_{L1}(230^{\circ}\mathrm{C}, 72 \text{ bar}) = 991.1$ kJ/kg.
- Outlet enthalpy: $h_{V2}(288^{\circ}\mathrm{C}, 72 \text{ bar}) = 2771$ kJ/kg.

$$P = M_L \Delta h \approx 602 \times 1780\,10^3 = 1071 \text{ MW}$$

- To be compared to 1115 MW on the primary side.

STEAM IS A PERFECT GAS

- Steam density at 100°C, 1 bar, $\rho_V = 0,5897$ kg/m^3.
- Perfect gas approximation: $pV = RT$, $R = 8.316$ J/mol/K, $M = 18$ g/mol.

$$V = \frac{RT}{p} = 3.103\,10^{-2}\ \mathrm{m}^3, \quad \rho = \frac{M}{V} = 0.5801\ \mathrm{kg/m}^3.$$

- Steam at 288°C, 72 bar, $\rho_V = 37.64$ kg/m^3.
- Perfect gas : $pV = RT$, $R = 8.316$ J/mol/K, $M = 18$ g/mol.

$$V = \frac{RT}{p} = 6.481\,10^{-4}\ \mathrm{m}^3, \quad \rho = \frac{M}{V} = 27.77\ \mathrm{kg/m}^3$$

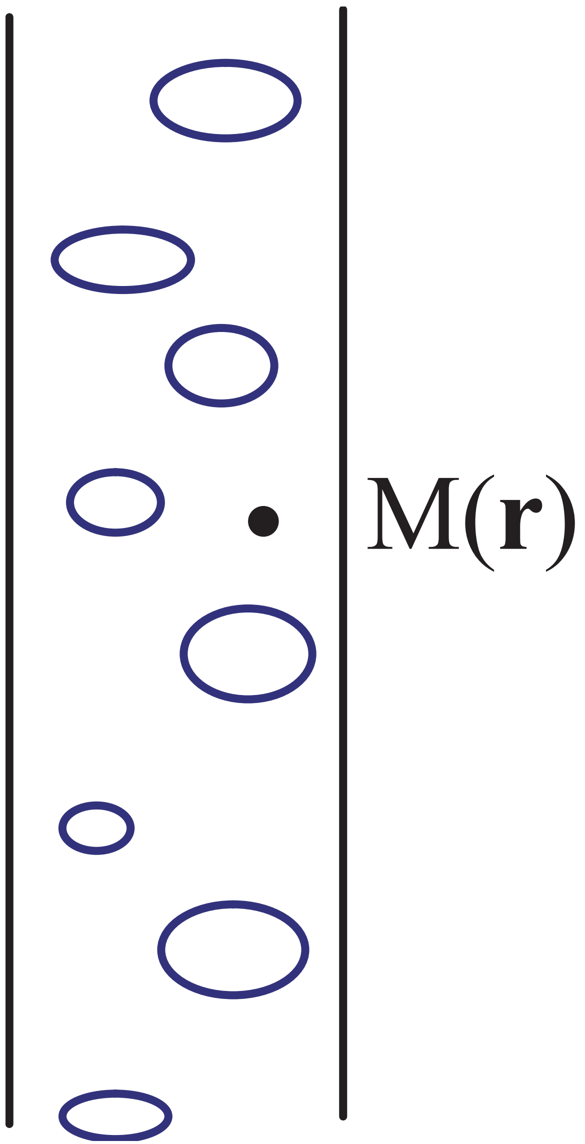
.

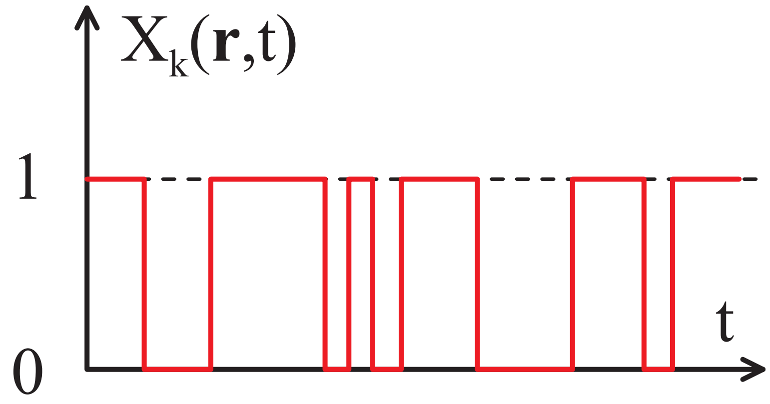
- Perfect gas approximation under-estimate by 26%.

TWO-PHASE FLOW VARIABLES

- Phase presence function
- Space averaging operators
- Instantaneous flow rates
- Time averaging operators
- Some mathematical properties
- Averaged flow rate and superficial velocity

PHASE PRESENCE FUNCTION





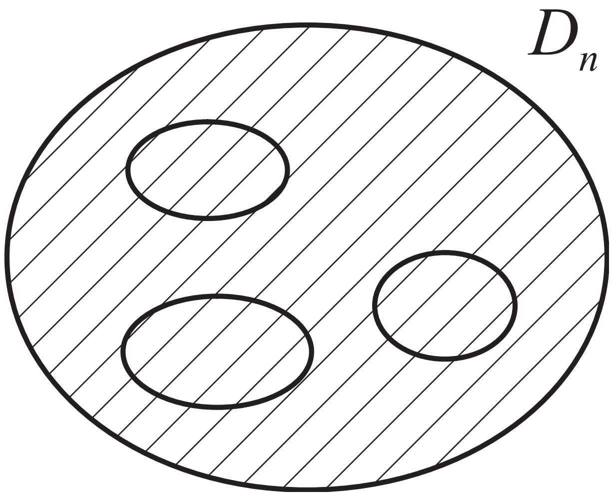
$$X_k(\mathbf{r},t) = \begin{cases} 1 & \text{if } x \in \text{ phase } k \\ 0 & \text{if } x \notin \text{ phase } k \end{cases}$$

Measurable variable,

- Resistive probe (electrical impedance)
- Optical probe (refraction index)
- Thermal anemometry (heat transfer)

Subscripts : $k = 1, 2$, $k = L, G$, $k = f, v$ *etc.*

SPACE AVERAGING (1/3)

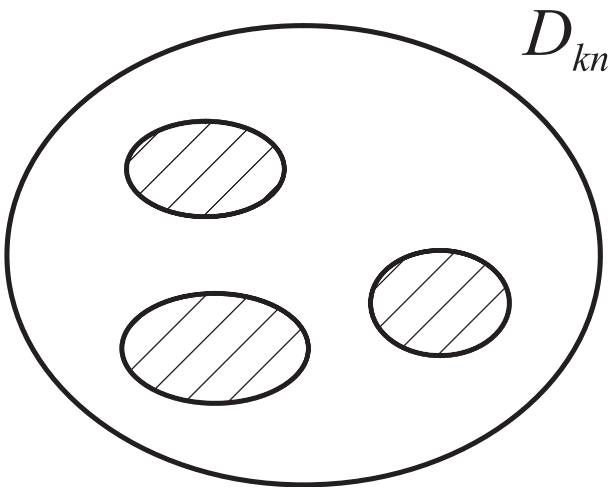


Space averaging operator (plain space average)

$$\langle\!\langle f \rangle\!\rangle_n \triangleq \frac{1}{D_n} \int_{D_n} f \, \mathrm{d}D_n$$

- n=1, line (chord in a pipe)
- n=2, surface (cross section)
- n=3, volume (some length of a pipe)

SPACE AVERAGING (2/3)

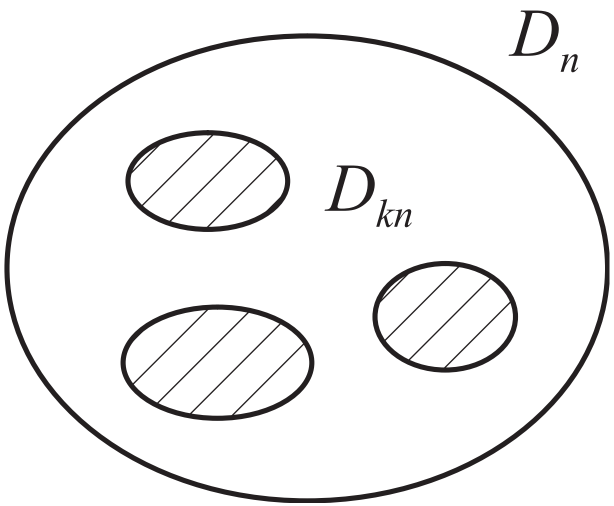


Phase presence conditional average,

$$< f_k >_n \triangleq \frac{1}{D_{kn}} \int_{D_{kn}} f_k \, \mathrm{d}D_{kn}$$

- n=1, line
- n=2, surface (shown here)
- n=3, volume

SPACE AVERAGING (3/3)



Instantaneous phase fraction

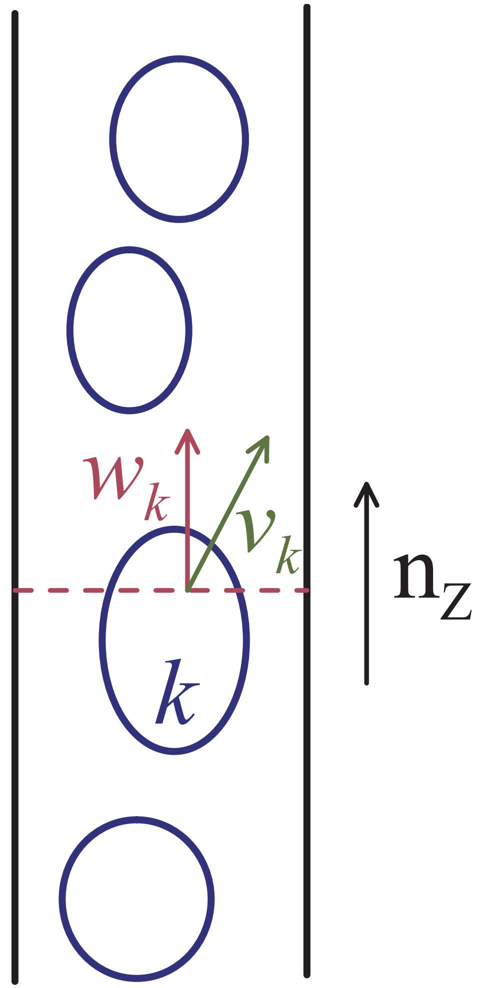
$$R_{kn}(t) \triangleq \nless X_k(\mathbf{r},t) \ngtr_n = \frac{D_{kn}}{D_n}$$

- n=1, line fraction, $\dfrac{L_k}{L_1 + L_2}$
- n=2, surface fraction, $\dfrac{A_k}{A_1 + A_2}$
- n=3, volume fraction, $\dfrac{V_k}{V_1 + V_2}$

Identity (proof left as an exercise)

$$R_{kn} < f >_{kn} = \nless X_k f \ngtr_n$$

FLOW RATE AND MASS FLOW RATE



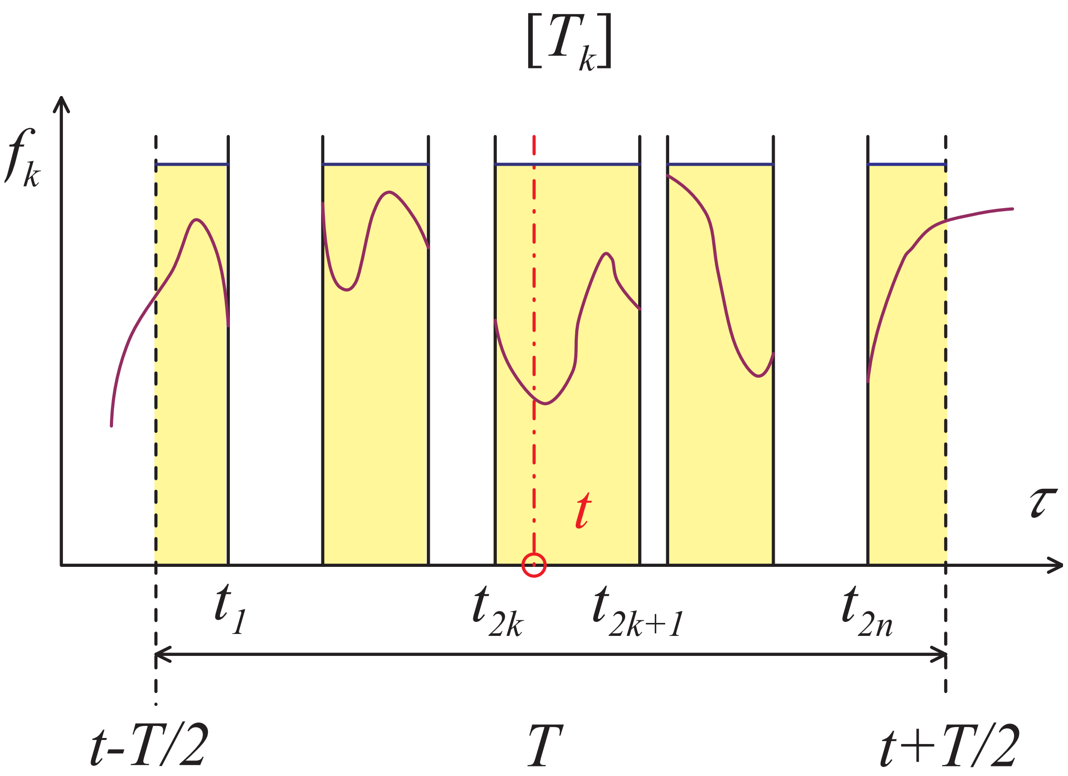
- Instantaneous flow rate (m^3/s), $w_k = \mathbf{v}_k \cdot \mathbf{n}_z$

$$Q_k(t) \triangleq \int_{A_k} w_k \, \mathrm{d}A_k = A_k < w_k >_2$$

- Instantaneous mass flow rate (kg/s)

$$M_k(t) \triangleq \int_{A_k} \rho_k w_k \, \mathrm{d}A_k = A_k < \rho_k w_k >_2$$

TIME AVERAGING



Time averaging on $[T]$, (plain)

$$\overline{f}(t) \triangleq \frac{1}{T}\int_{t-T/2}^{t+T/2} f(\tau)\,\mathrm{d}\tau$$

Conditional time averaging, $[T_k]$

$$\overline{f}_k^X(t) \triangleq \frac{1}{T_k}\int_{[T_k]} f_k(\tau)\,\mathrm{d}\tau$$

Local time fraction, *void fraction* for gas/vapor

$$\alpha_k(\mathbf{r},t) \triangleq \frac{T_k}{T} = \overline{X_k(\mathbf{r},t)}$$

Identity, derives from the definitions,

$$\alpha_k \overline{f}^X = \overline{X_k f}$$

COMMUTATIVITY OF AVERAGING OPERATORS

$$\overline{R_{kn} < f_k >_n} = \lessdot \alpha_k \overline{f_k}^X \gtrdot_n$$

Proof, from definitions:

$$\overline{R_{kn} < f_k >_n} = \frac{1}{T} \int_{[T]} \left\{ \frac{R_{kn}}{D_{kn}} \int_{D_{kn}(t)} f_k \, \mathrm{d}D_{kn} \right\} \mathrm{d}t$$

$$\frac{1}{T} \int_{[T]} \mathrm{d}t \frac{1}{D_n} \int_{D_n} X_k f_k \, \mathrm{d}D_n = \frac{1}{D_n} \int_{D_n} \mathrm{d}D_n \frac{1}{T} \int_{[T]} X_k f_k \, \mathrm{d}t$$

$$\frac{1}{D_n} \int_{D_n} \left\{ \frac{\alpha_k(\mathbf{r})}{T_k} \int_{[T_k]} f_k \, \mathrm{d}t \right\} \mathrm{d}D_n = \lessdot \alpha_k \overline{f_k}^X \gtrdot_n$$

Significant example: mean void fraction, $f_k = 1$, salami theorem...

$$\overline{R_{kn}} = \lessdot \alpha_k \gtrdot_n$$

MEAN FLOW RATES

- Mean volume flow rate,

$$\overline{Q_k} = A\overline{R_{k2} < w_k >_2} = A \nless \alpha_k \overline{w}_k^X \ngtr_2$$

- Mean mass flow rate,

$$\overline{M_k} = A\overline{R_{k2} < \rho_k w_k >_2} = A \nless \alpha_k \overline{\rho_k w}_k^X \ngtr_2$$

SUPERFICIAL VELOCITY

- Mean volumetric flux,

$$j_k \triangleq \overline{X_k w_k} \equiv \alpha_k \overline{w}_k^X$$

- Mean mass flux,

$$g_k \triangleq \overline{X_k \rho_k w_k} \equiv \alpha_k \overline{\rho_k w}_k^X$$

- Superficial velocity (*vitesse débitante*),

$$J_k = \langle j_k \rangle_2 = \langle \alpha_k \overline{w}_k^X \rangle_2 = \frac{\overline{Q_k}}{A}$$

- Mixture superficial velocity,

$$J = J_1 + J_2 = \frac{\overline{Q_1} + \overline{Q_2}}{A}$$

QUALITY

- Mean mass flux,

$$G_k \triangleq \langle\!\langle g_k \rangle\!\rangle_2 = \langle\!\langle \alpha_k \overline{\rho_k w_k}^X \rangle\!\rangle_2 = \frac{\overline{M_k}}{A}$$

- Mixture mean mass flux,

$$G = G_1 + G_2 = \frac{\overline{M_1} + \overline{M_2}}{A}$$

- Quality, (*titre massique*),

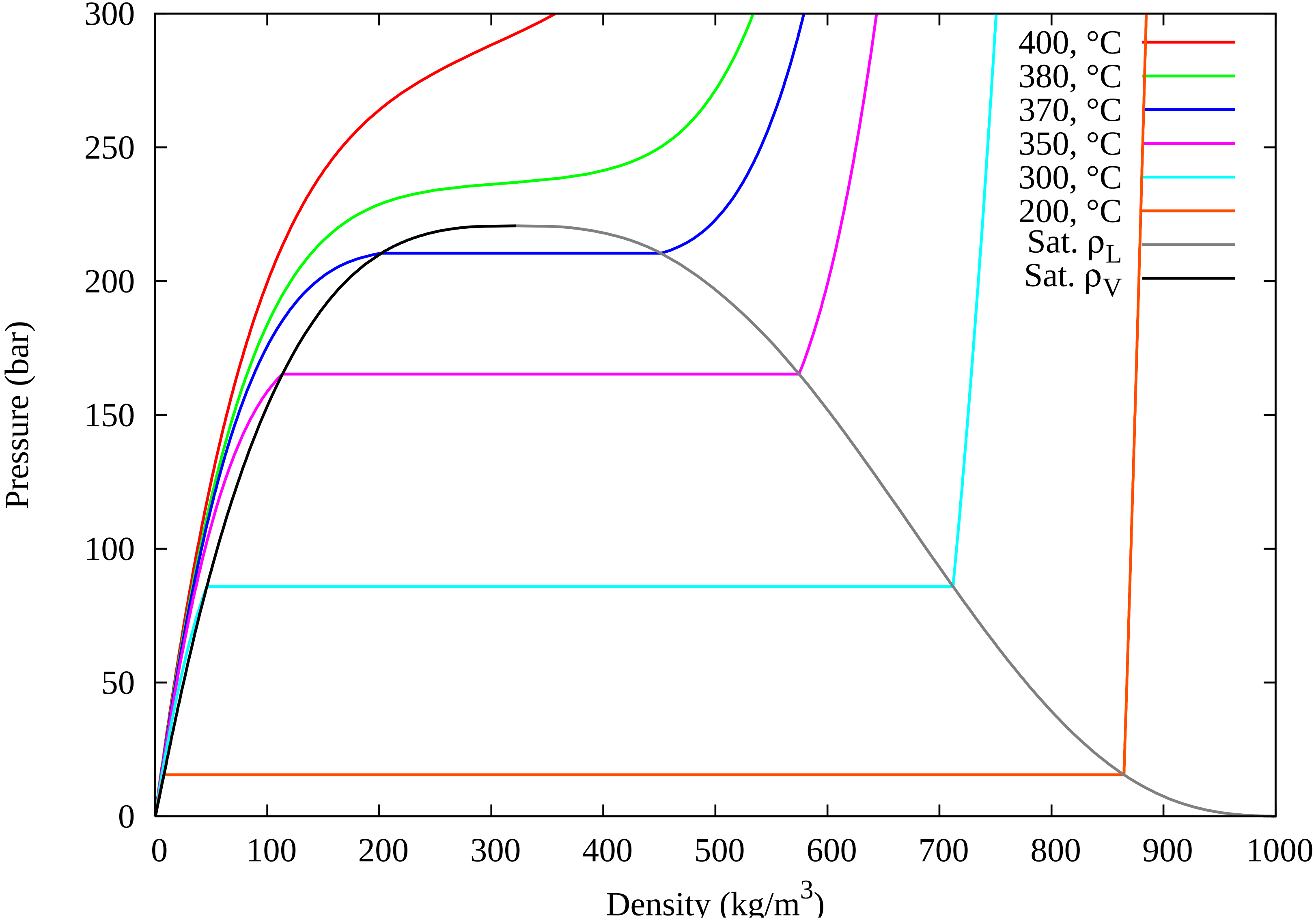
$$x_k = \frac{M_k}{M}, \quad M = M_1 + M_2$$

- Volume quality,

$$\beta_k = \frac{Q_k}{Q}, \quad Q = Q_1 + Q_2$$

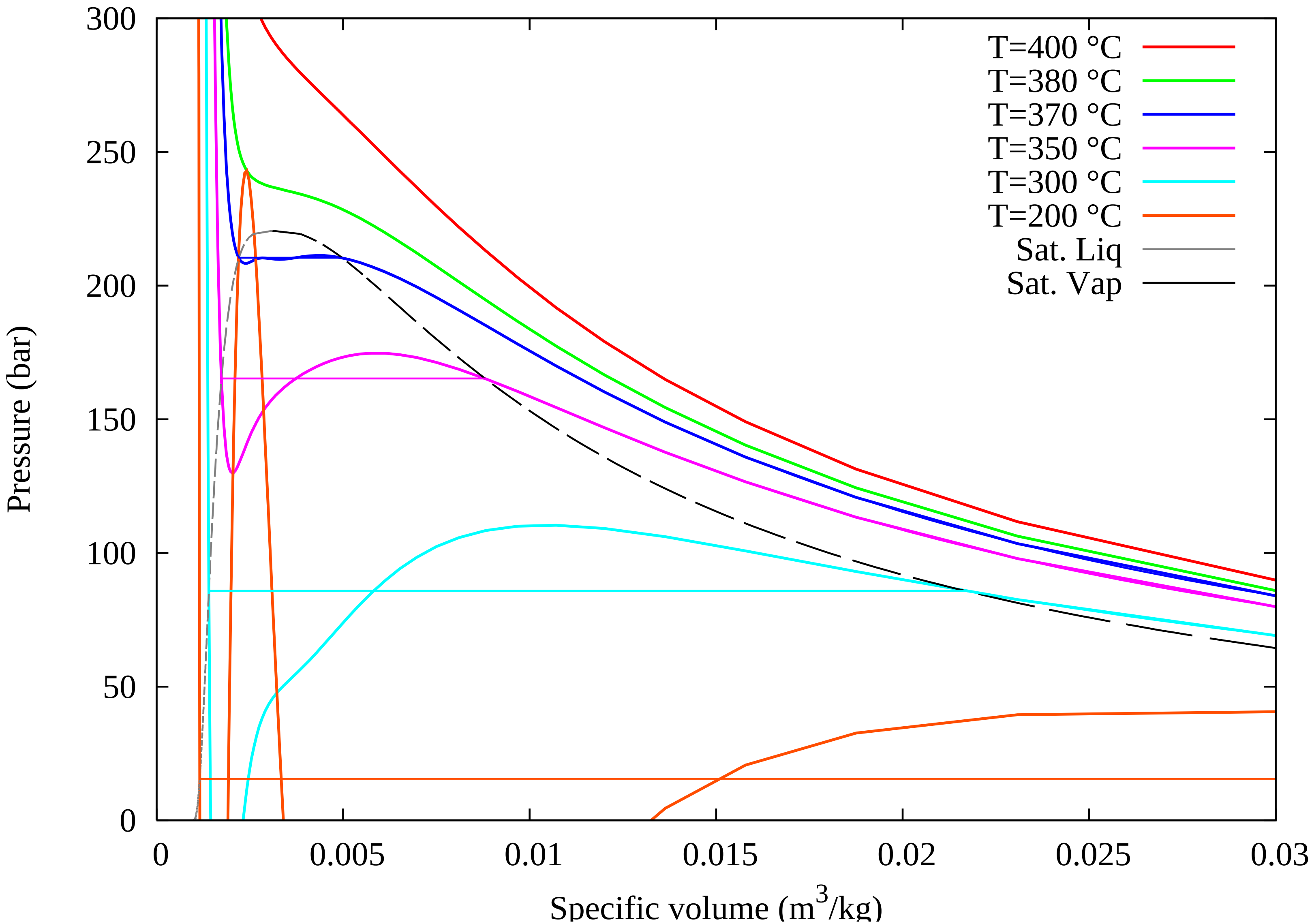
- Equilibrium (steam) quality...

WATER THERMODYNAMIC DIAGRAM (ρ, p)

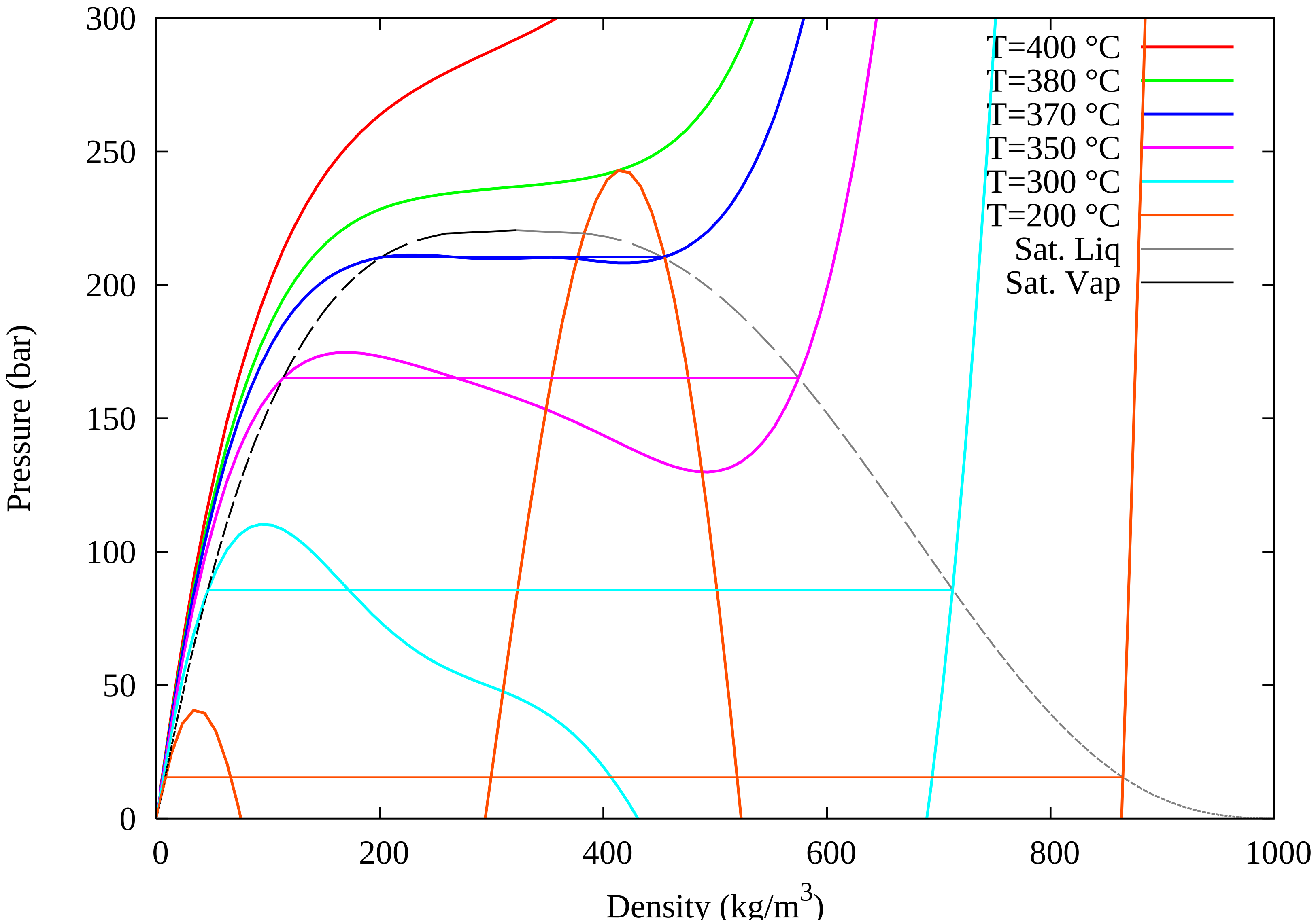


Critical pressure and temperature : $\approx$ 221 bar, 373,9°C.

STEAM TABLES AND EOS (v, p)



STEAM TABLES AND EOS (ρ, p)



EQUILIBRIUM QUALITY

- The thermodynamic equilibrium assumption,

$$T_L = T_V = T_{\text{sat}}(p), \quad h_k(T_k, p) = h_k(T_{\text{sat}}(p), p) \triangleq h_{k\text{sat}}(p)$$

- 1D model assumption, flat profiles ($h_k(\mathbf{r}) \neq < k_k >_2$),
- Energy balance, q, uniform heat flux distribution

$$P = \pi q D z = M[h(z) - h_1] = M[(x_{eq} h_{V\text{sat}} + (1 - x_{eq}) h_{L\text{sat}}) - h_{L1})]$$

$$x_{eq} = \frac{h_{L1} - h_{L\text{sat}}}{h_{lv}} + \frac{\pi q D z}{M h_{lv}}$$

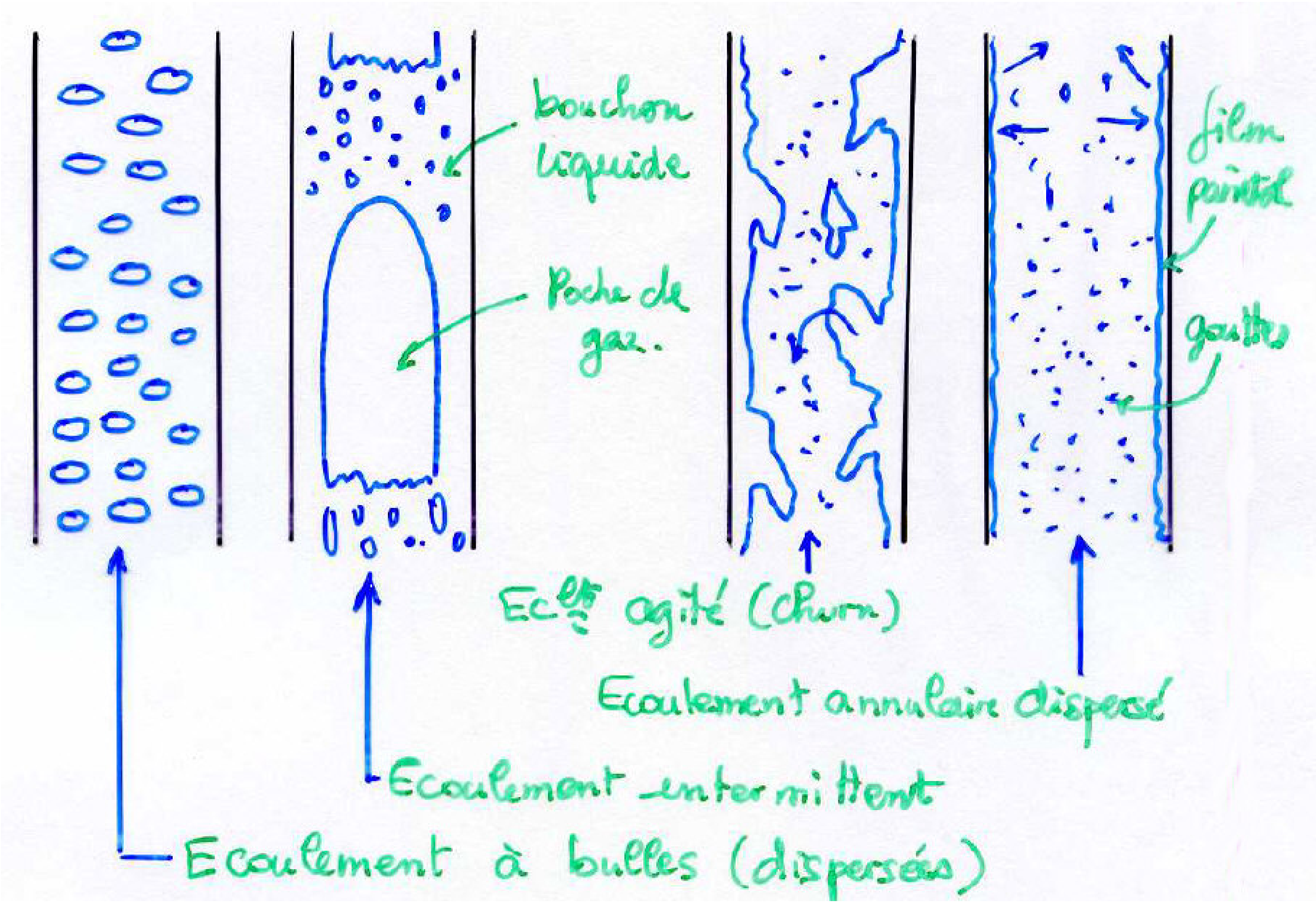
- Phase change enthalpy : $h_{lv} \triangleq h_{V\text{sat}} - h_{L\text{sat}}$
- Equilibrium quality varies linearly with position.
- h is the mean enthalpy (energy balance), non dimensional enthalpy,

$$\boxed{x_{eq} = \frac{h - h_{L\text{sat}}}{h_{lv}}}$$

FLOW REGIMES

- Topological phase organisation in flows,
  - Bubbles, *bulles*
  - Plugs and slugs, *poches et bouchons*
  - Liquid films, drops and droplets
  - No sharp transitions
- Modeling is the motivation for identification of flow regimes
  - Single-phase flows: laminar-turbulent (NS or RANS)
  - Two-phase flows: structures of interfaces $\rightarrow$ model.
  - Shortcomings: fully developed flows, fuzzy transitions, hydrodynamic singularities.
- Control variables: flow rates, slope, direction, diameter, transport properties, inlet conditions *etc.*
- Examples: vertical and horizontal co-current flows.

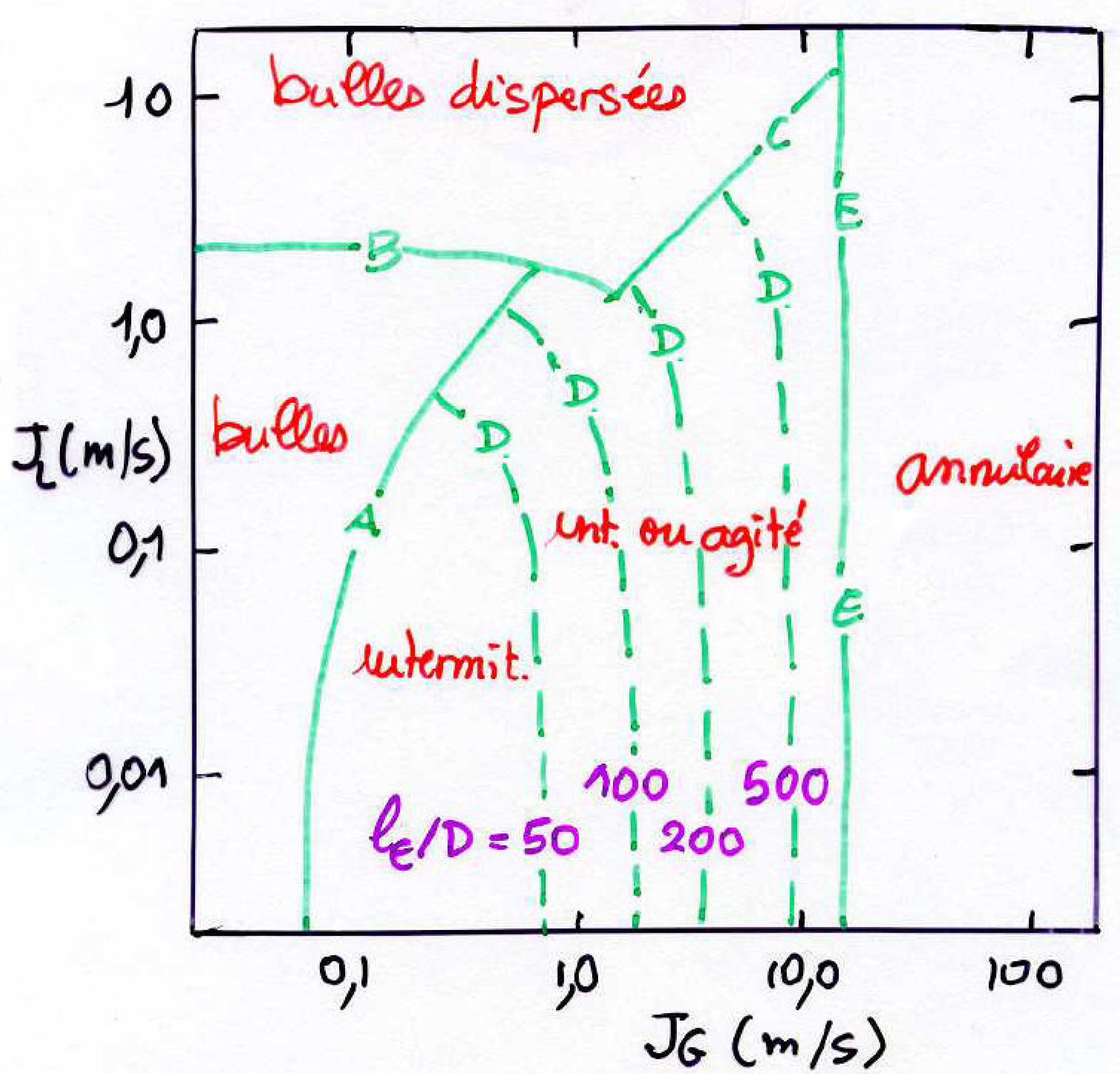
VERTICAL ASCENDING FLOWS



Flow regime transitions:

- Experiments, empirical, flow rates, momentum fluxes.
- Transition modeling, mechanisms, (Dukler & Taitel, 1986).

TAITEL ET DUKLER (1980) MODEL



Vertical ascending air-water flow, $D = 50$ mm, $P = 1$ bar.

Flow regimes:

- A-B: Bubbly (*bulles*)
- A-D:Intermittent (*poches, bouchons*)
- D-E: Churn (*agité*)
- E: Annular
- B-C: Dispersed bubbles

TAITEL ET DUKLER (1980) MODEL

- Bubbly flow and intermittent (A): Bubble coalescence, zig-zag motion of bubbles.

$$J_L = \frac{1-\alpha}{\alpha} J_G - (1-\alpha)^{\frac{3}{2}} U_{0\infty}, \quad \alpha_T = 0,25, \quad U_{0\infty} = 1,53 \left(\frac{g(\rho_L - \rho_G)\sigma}{\rho_L^2} \right)^{\frac{1}{4}}$$

- Dispersed bubbles and bubbly : turbulent break up, small bubbles, rectilinear path (B), dense packing (A) with $\alpha_T = 0,52$ (D).

$$\frac{2[\rho_L/(\rho_L - \rho_G)g]^{0,5} \nu_L^{0,08}}{(\sigma/\rho_L)^{0,10} D^{0,48}} J^{1,12} \geqslant 3,0$$

- Intermittent and churn (D): churn flow $\equiv$ development of slug flow.

$$\frac{L}{D} = 42,6 \left(\frac{J}{\sqrt{gD}} + 0,29 \right)$$

TAITEL ET DUKLER (1980) MODEL

- Annular (E): all liquid entrained by the gas, force balance,

$$\frac{J\rho_G^{\frac{1}{2}}}{[\sigma g(\rho_L - \rho_G)]^{\frac{1}{2}}} = 3,1$$

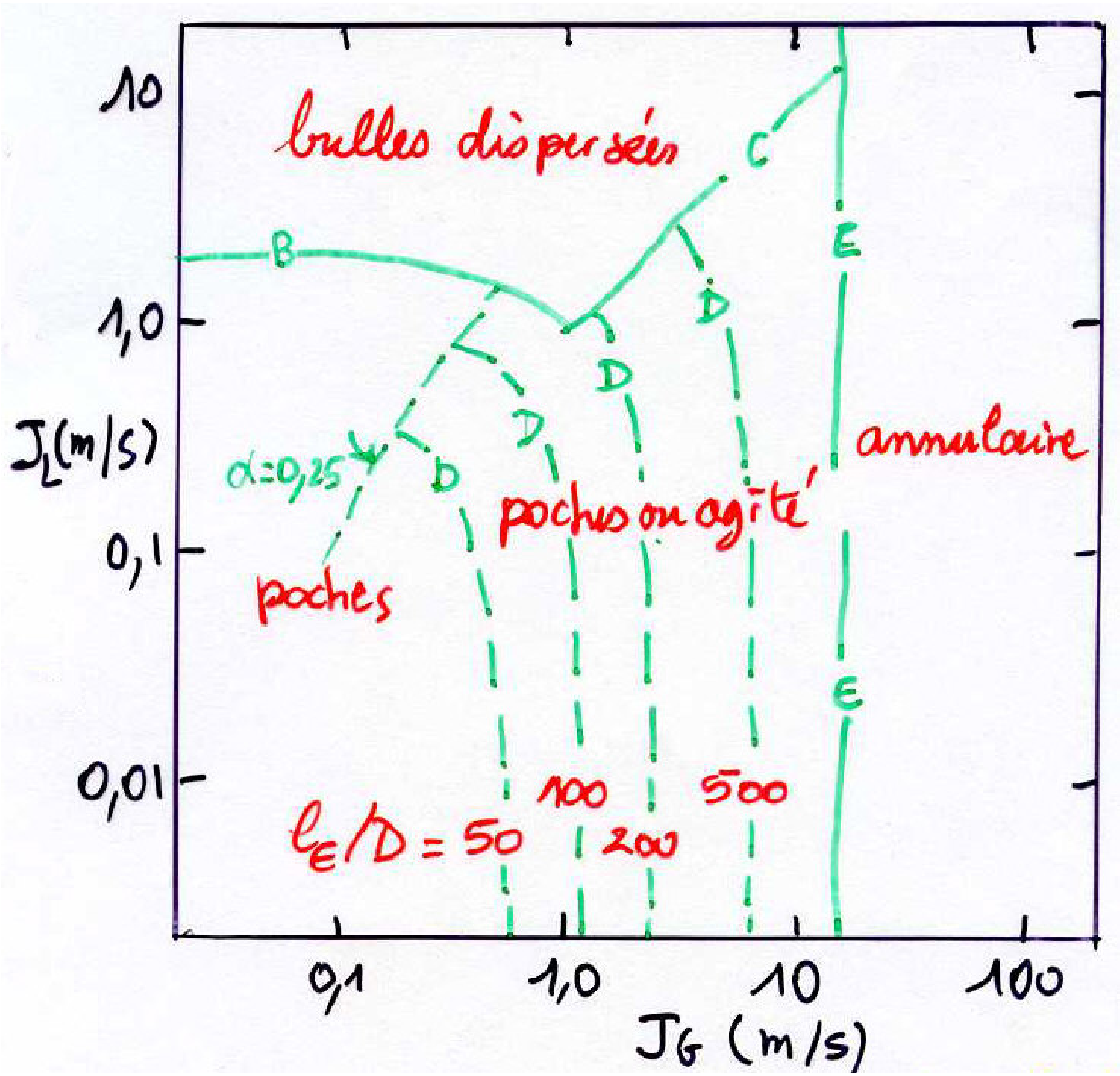
  See also flooding correlations & Ku.

- *Small* diameter pipes: bubbles (zig-zag) and occasionally Taylor bubbles.
  - Taylor bubbles relative velocity: $U_T = 0,35\sqrt{gD}$
  - individual bubbles: $U_B = U_{0\,\inf}(1-\alpha)^{\frac{1}{2}}$
  - In small diameter pipes, Taylor bubbles are slower than individual bubbles *to* coalescence towards slugs.

$$\left(\frac{\rho_L^2 dD^2}{(\rho_L - \rho_G)\sigma}\right)^{\frac{1}{4}} \leqslant 3,78$$

  - Bubbly flow no longer exists.

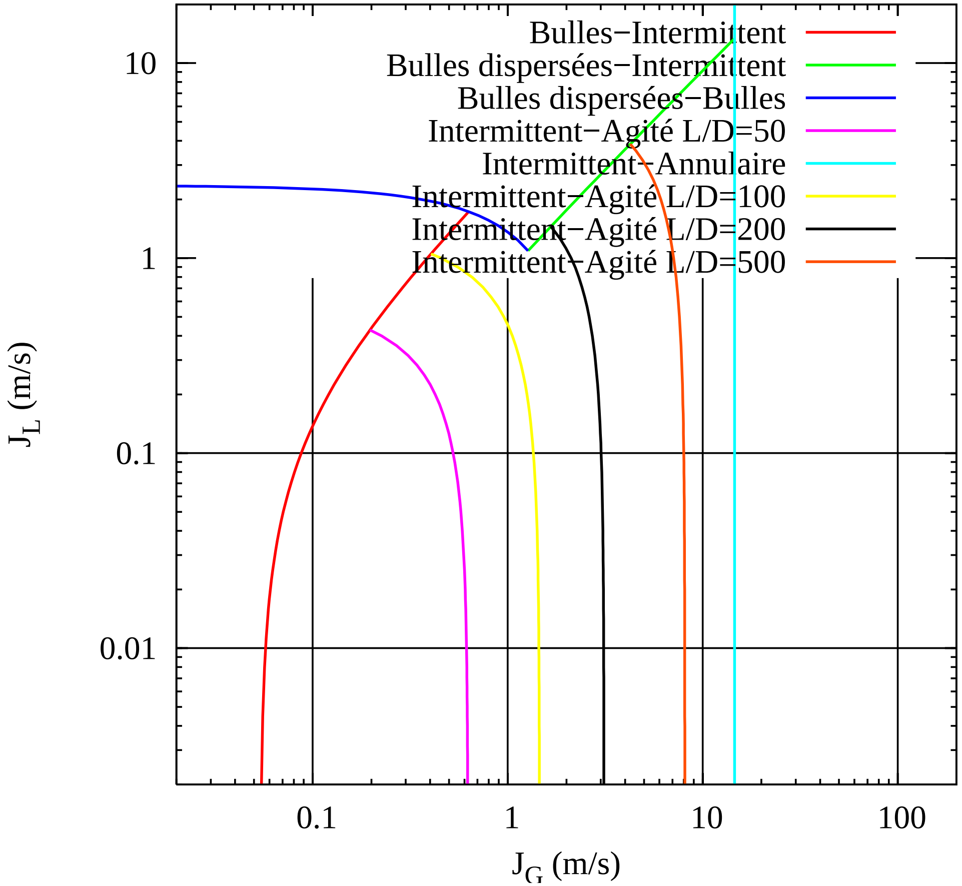
TAITEL ET DUKLER (1980) MODEL



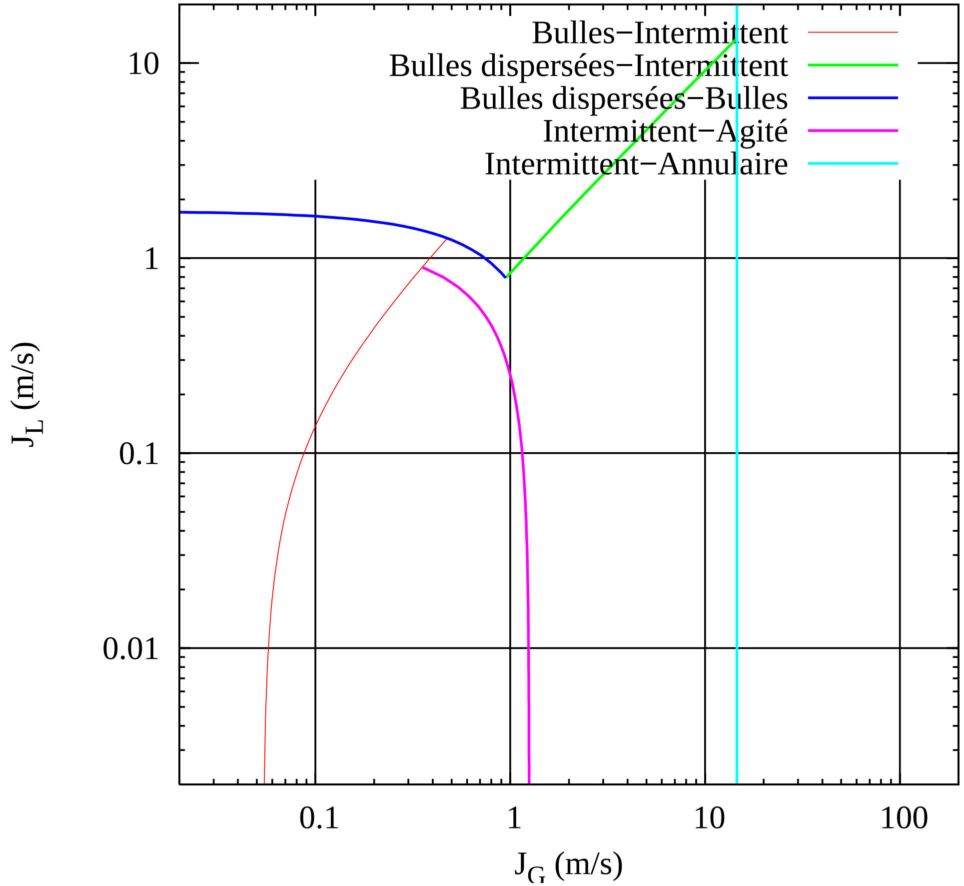
Air water, $D = 25$ mm, $P = 1$ bar.

Small diameter pipes: no bubbly flow.

APPLICATIONS : `VertTD02`

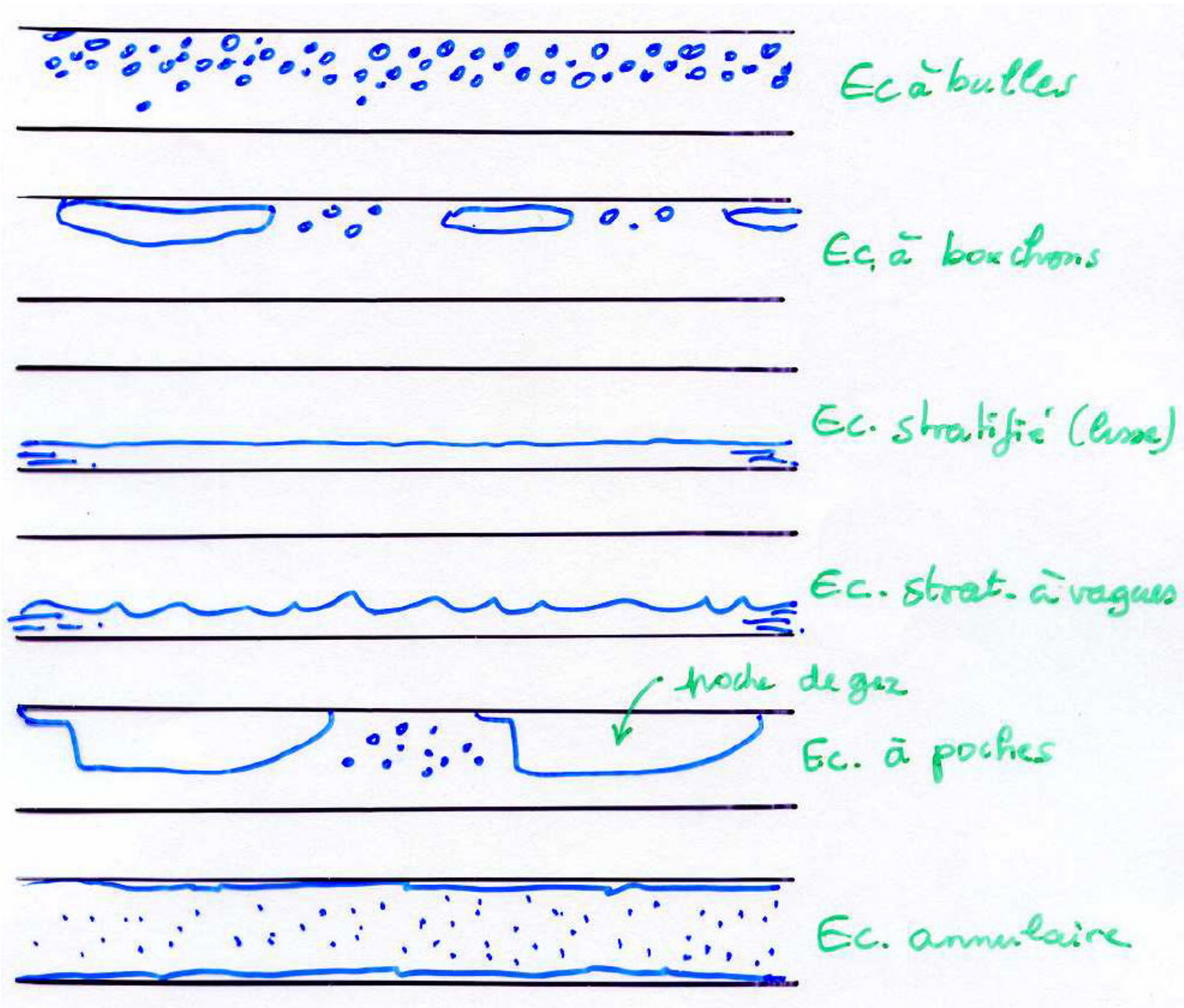


Air-water, 51 mm, 1 bar



Air-water, 25 mm, $L/D = 100$, 1 bar

FLOW PATTERNS IN HORIZONTAL FLOW



Main flow regimes:

- Bubbly
- Plug
- Stratified, smooth or wavy
- Slug of gas and plugs of liquid
- Annular

Modeling the transition based on mechanisms (Dukler & Taitel, 1986).

FLOW PATTERN IN HORIZONTAL FLOW

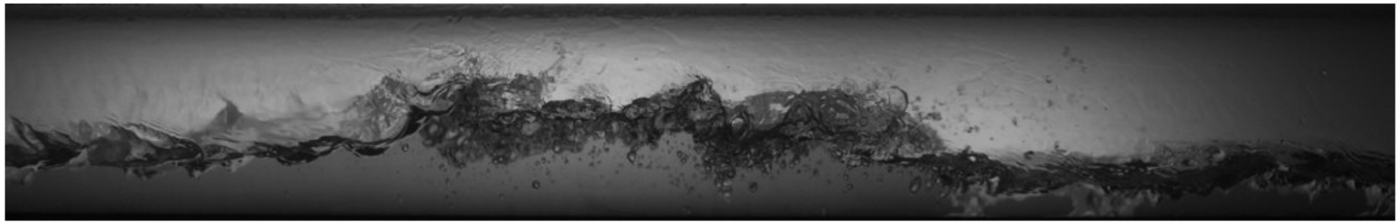


Photo 17, Wavy-stratified flow, with roll waves, Test 33

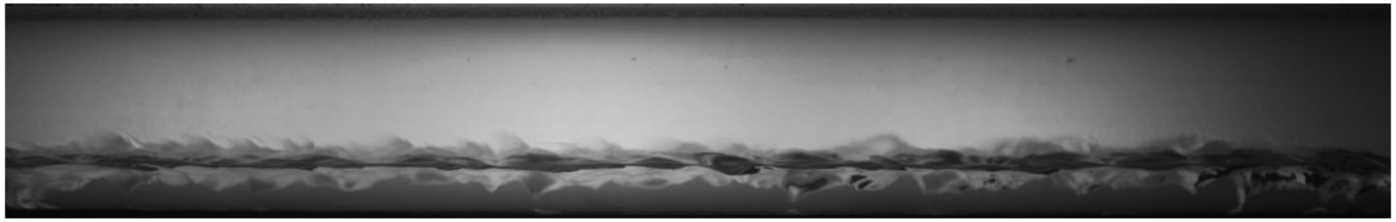


Photo 15 , Wavy-stratified flow, Test 32.

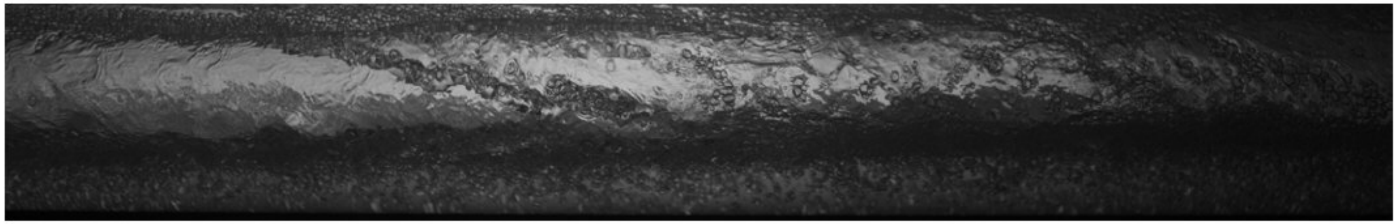


Photo 25, Slug flow.

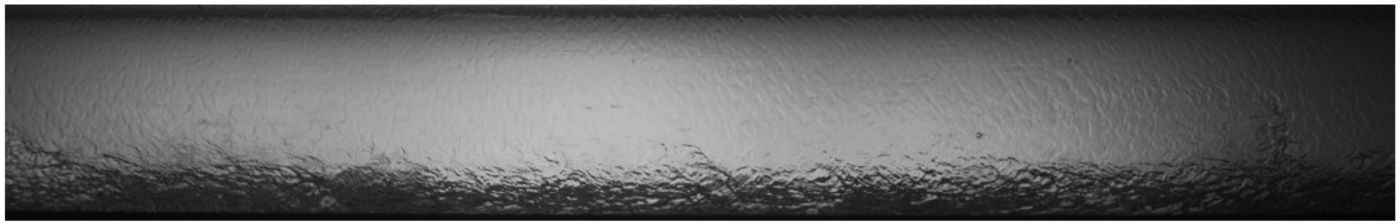


Photo 22, Annular flow.

HORIZONTAL SLUG FLOW

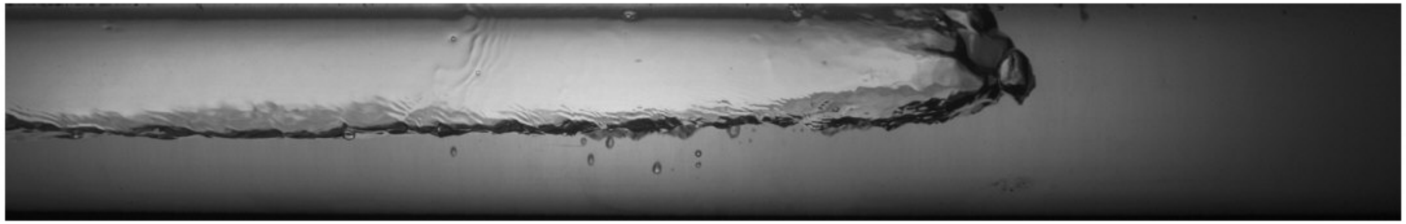


Photo 02.

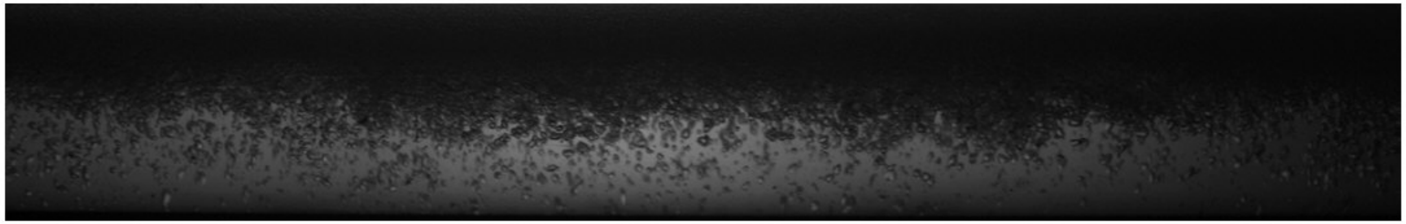


Photo 07.

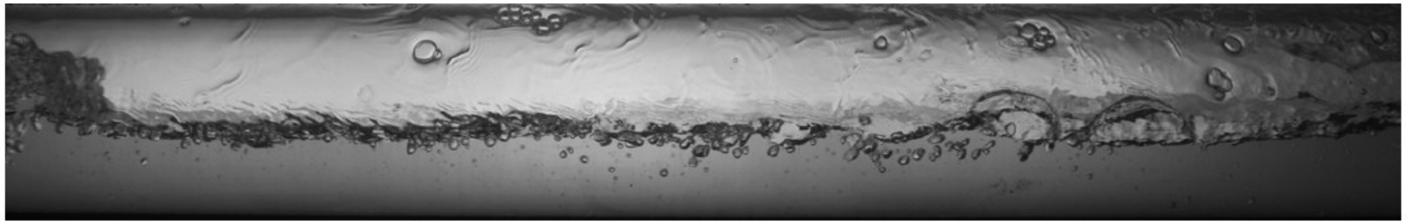


Photo 09.

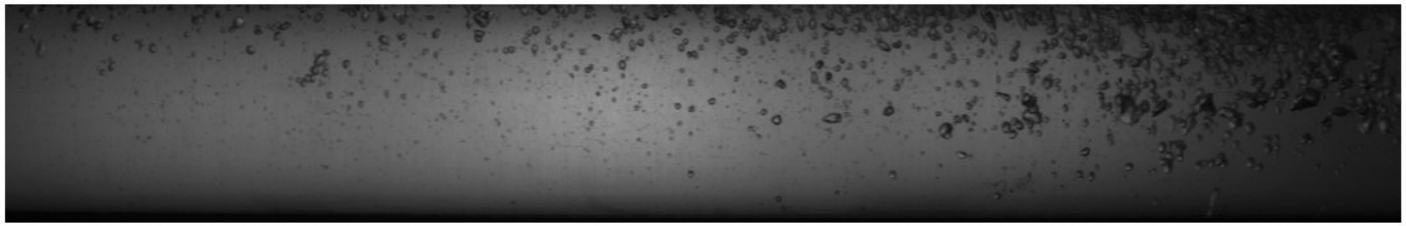
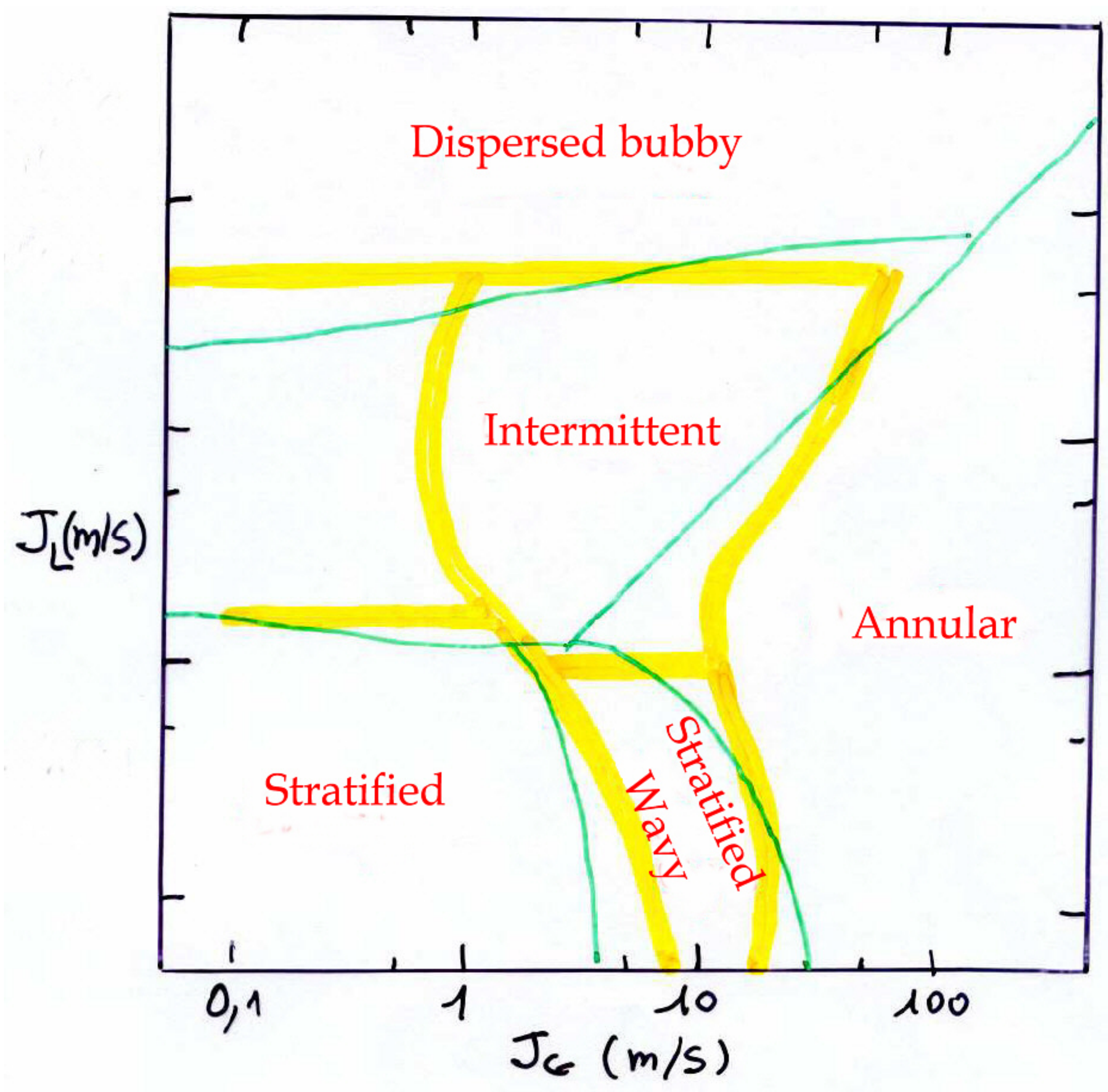


Photo 12.

$$\alpha = 22\%$$

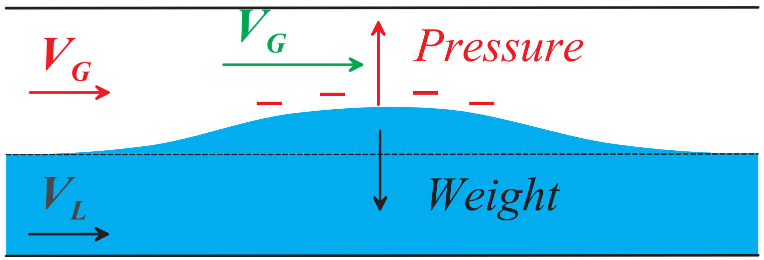
DUKLER AND TAITEL (1976) MODEL

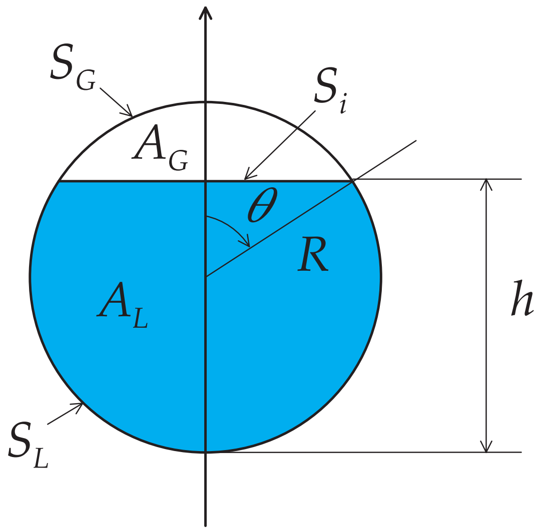


Transition criteria:

- Empirical, e.g.: Mandhane, air-water, 25 mm, 1 bar.
- Mechanistic modeling of transition (Dukler & Taitel, 1986).

TAITEL AND DUKLER (1976) MODEL





- Stability of stratified flow, linear stability analysis (back later on, see also exercises)
- The Kelvin-Helmholtz instability,

$$V_G \geqslant C_2 \left[\frac{(\rho_L - \rho_G)\cos\beta A_G}{\rho_G \frac{\mathrm{d}A_L}{\mathrm{d}h}} \right]^{\frac{1}{2}}, \quad C_2 \approx 1 - \frac{h}{D}$$

- Base flow: smooth an horizontal interface

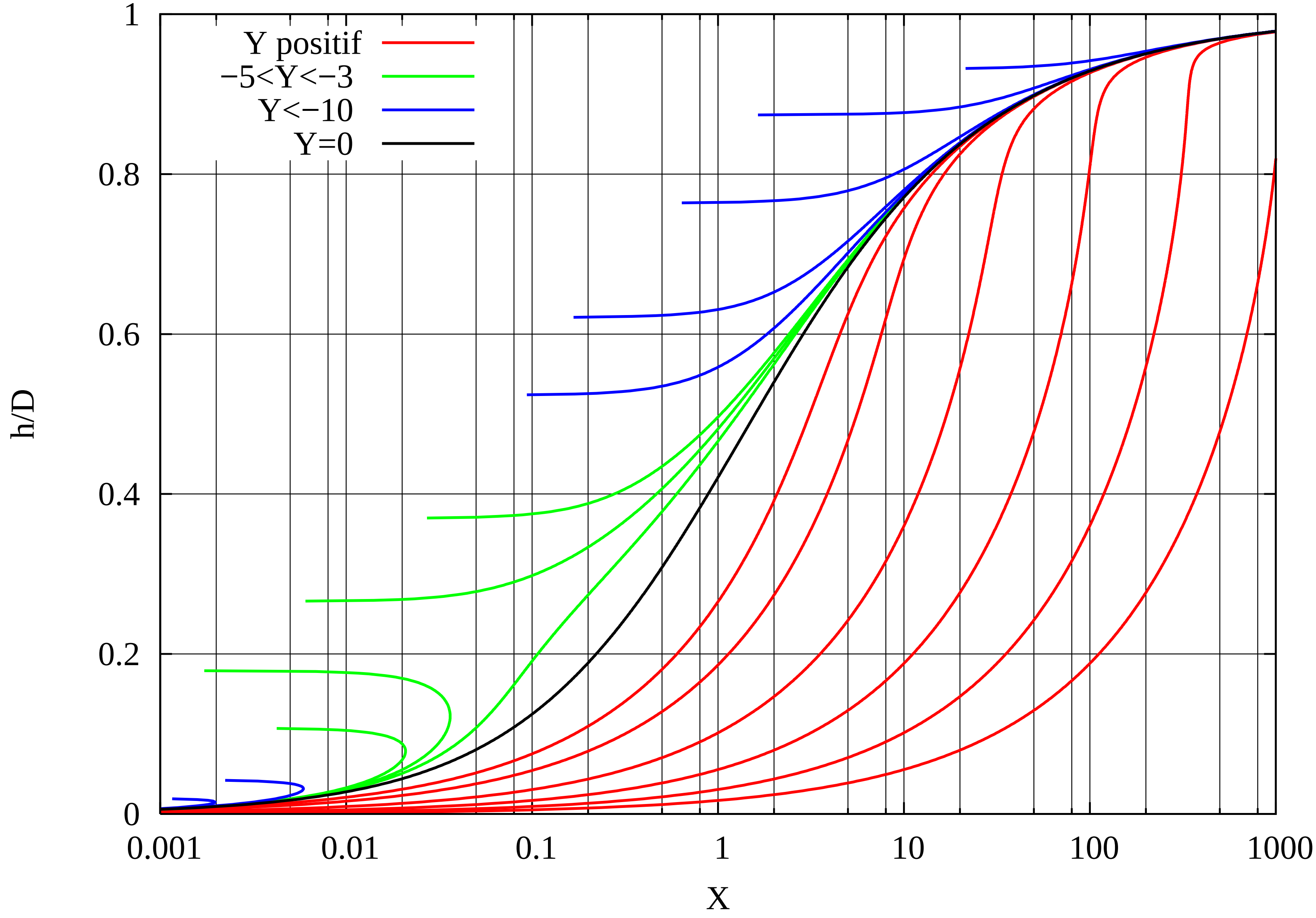
$$\tau_G \frac{S_G}{A_G} - \tau_L \frac{S_L}{A_L} + \tau_i \left(\frac{S_i}{A_L} + \frac{S_i}{A_G} \right) + (\rho_L - \rho_G) g \sin\beta = 0$$

$$X^2 f(A, D, A_L, P_L, D_L) - g(A, D, A_G, P_G, D_G, P_i) - 4Y$$

$$X^2 = \frac{\frac{1}{2} C_L \rho_L J_L^2 Re_{LS}^{-n}}{\frac{1}{2} C_G \rho_G J_G^2 Re_{GS}^{-n}} = \left| \frac{(\mathrm{d}P/\mathrm{d}z)_{LS}}{(\mathrm{d}P/\mathrm{d}z)_{GS}} \right|.$$

$$Y = \frac{(\rho_L - \rho_G) g \sin\beta}{\frac{4}{D} \frac{1}{2} \rho_G J_G^2 C_G Re_G^{-m}}$$

TAITEL AND DUKLER (1976) MODEL



TAITEL AND DUKLER (1976) MODEL

- Transition, stratified flow instability:

$$F^2 \left(\frac{1}{C_2^2} \frac{\tilde{U}_G^2 \frac{\mathrm{d}\tilde{A}_G}{\mathrm{d}\tilde{h}}}{\tilde{A}_G} \right) \geqslant 1, \quad \tilde{U}_G = \frac{A}{A_G}, \quad F = \left(\frac{\rho_G}{\rho_G - \rho_L} \right)^{\frac{1}{2}} \frac{J_G}{(Dg\cos\beta)^{\frac{1}{2}}}$$

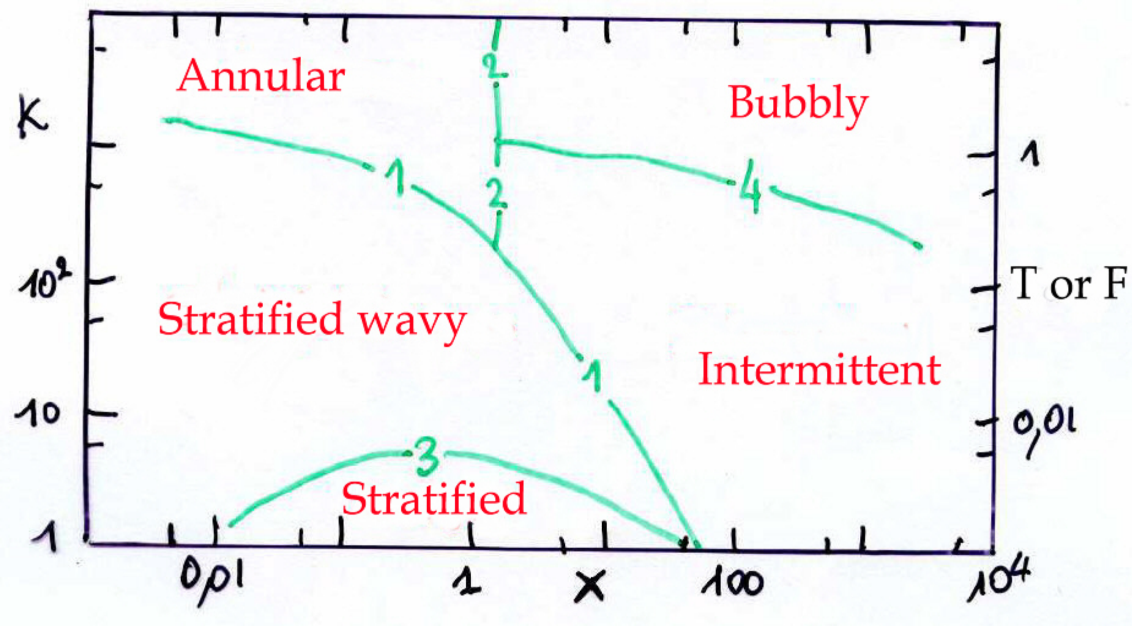
- Towards intermittent flow: $\frac{h}{D} \geqslant 0,5$
- Towards annular flow: $\frac{h}{D} \leqslant 0,5$
- Stratified smooth or wavy ?:

$$U_G \geqslant \left(\frac{4\nu_L(\rho_L - \rho_G) g\cos\beta}{s\rho_G U_L} \right)^{\frac{1}{2}}, \; K \geqslant \frac{2}{\tilde{U}_G \sqrt{s\tilde{U}_L}}, \; K^2 = \left(\frac{\rho_G J_G^2}{(\rho_L - \rho_G) Dg\cos\beta} \right) \left(\frac{DJ_L}{\nu_L} \right)$$

- Dispersed bubbles:

$$U_L \geqslant \left(\frac{4A_G}{S_i} \frac{g\cos\beta}{f_L} \left(1 - \frac{\rho_G}{\rho_L} \right) \right)^{\frac{1}{2}}, \; T^2 \geqslant \frac{8\tilde{A}_G}{\tilde{S}_i \tilde{U}_L^2 (\tilde{U}_L \tilde{D}_L)^{-n}}, \; T = \left(\frac{\left| \frac{\mathrm{d}p}{\mathrm{d}z} \right|_{LS}}{(\rho_L - \rho_G) g\cos\beta} \right)^{\frac{1}{2}}$$

TAITEL AND DUKLER (1976) MODEL



Curves	1 et 2	3	4
Coordinates	F, X	K, X	T, X

$$K = \left(\frac{\rho_G J_G^2}{(\rho_L - \rho_G) D g \cos\beta}\right)^{\frac{1}{2}} \left(\frac{D J_L}{\nu_L}\right)^{\frac{1}{2}}$$

$$F = \left(\frac{\rho_G}{\rho_G - \rho_L}\right)^{\frac{1}{2}} \frac{J_G}{(D g \cos\beta)^{\frac{1}{2}}}$$

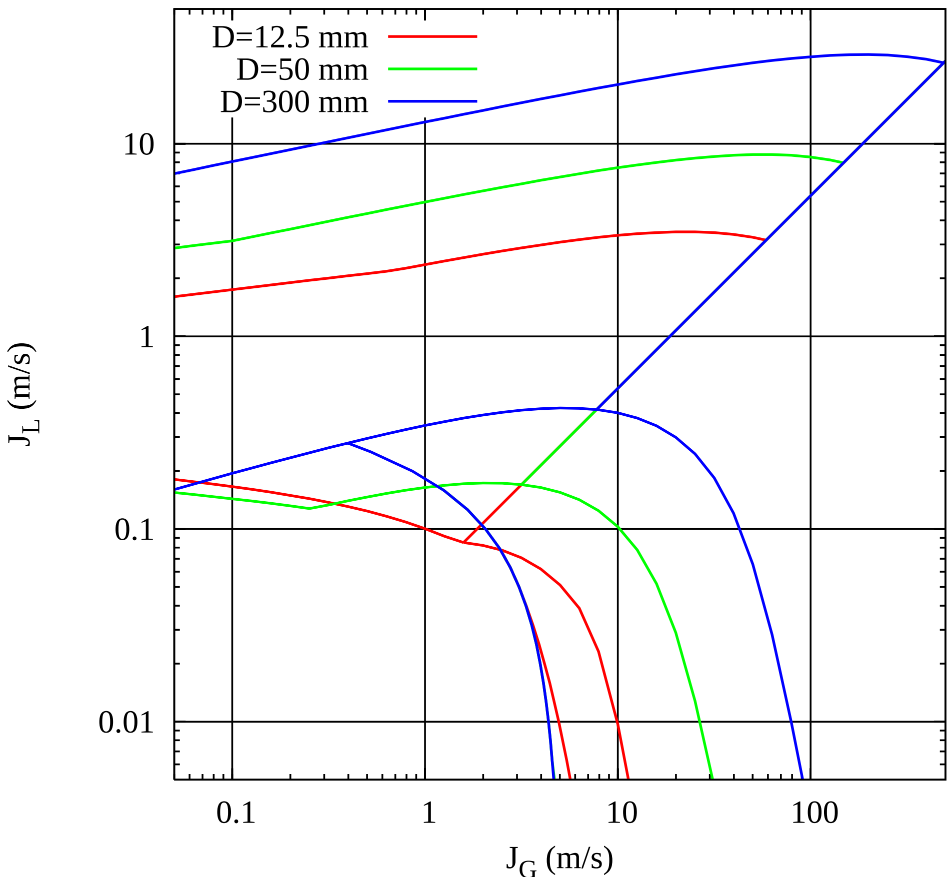
$$T = \left(\frac{\left|\frac{\mathrm{d}p}{\mathrm{d}z}\right|_{LS}}{(\rho_L - \rho_G) g \cos\beta}\right)^{\frac{1}{2}}$$

$$X = \left|\frac{(\mathrm{d}P/\mathrm{d}z)_{LS}}{(\mathrm{d}P/\mathrm{d}z)_{GS}}\right|^{\frac{1}{2}}$$

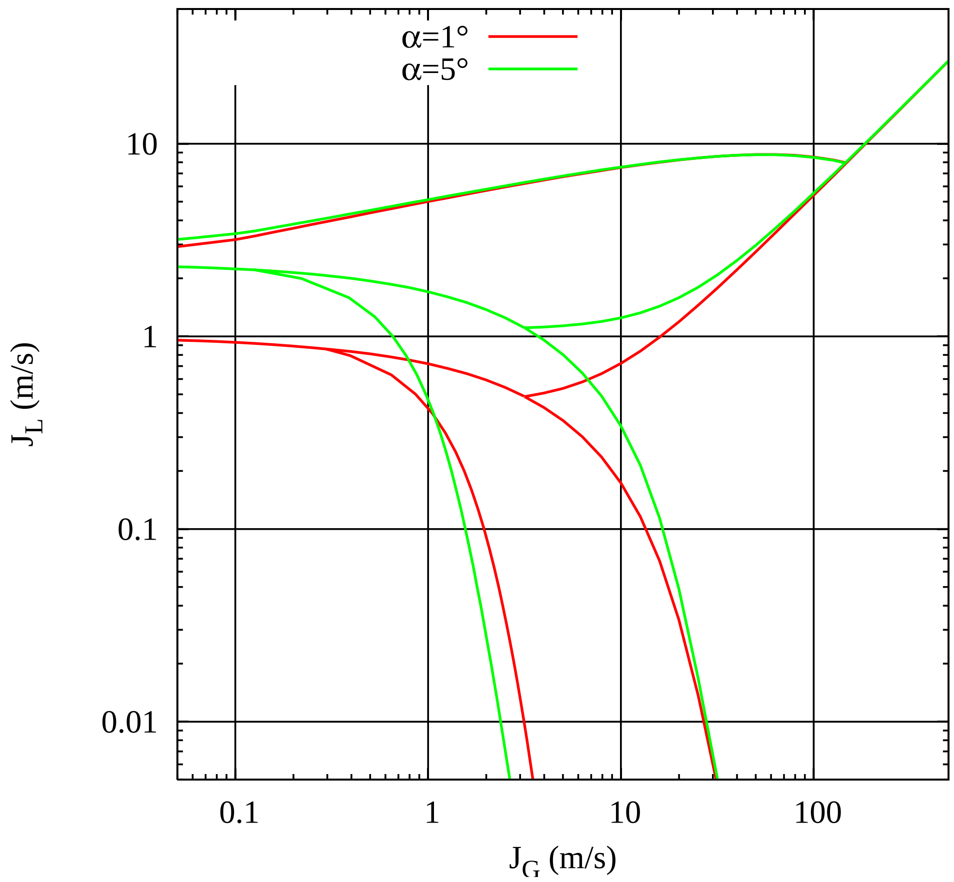
$$\left|\frac{\mathrm{d}p}{\mathrm{d}z}\right|_S = \frac{4C}{D}\mathrm{Re}^{-n}\frac{\rho J^2}{2}, \quad \mathrm{Re} = \frac{JD}{\nu}$$

- Laminar: $C = 16$, $n = 1$. Turbulent: $C = 0,046$, $n = 0,2$
- β: slope angle, $\beta = 0$, horizontal flow, $\beta > 0$, descending flows.

APPLICATIONS : `HoriTD03`

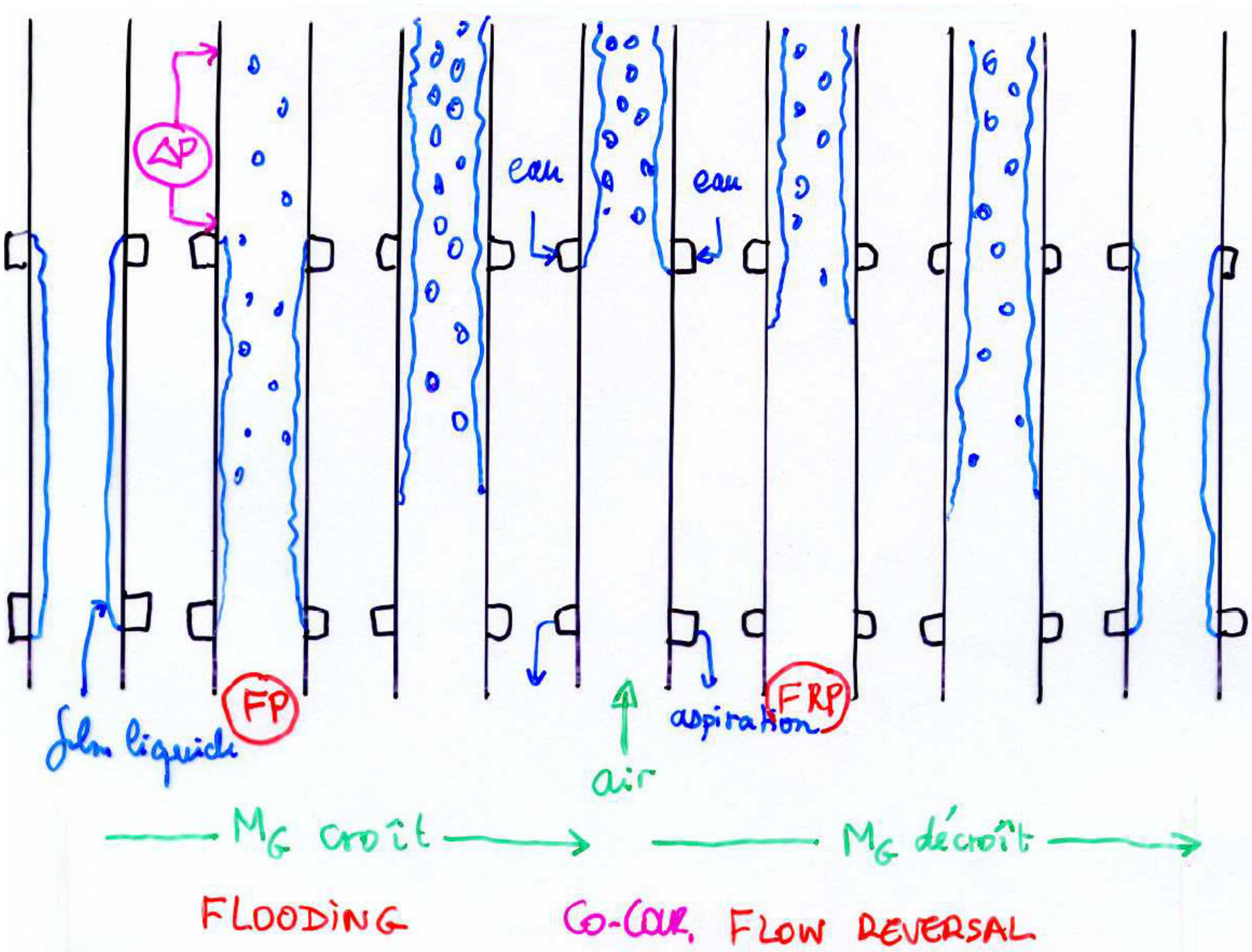


Air-Water, 50 mm, 1 bar, $\beta = 0$



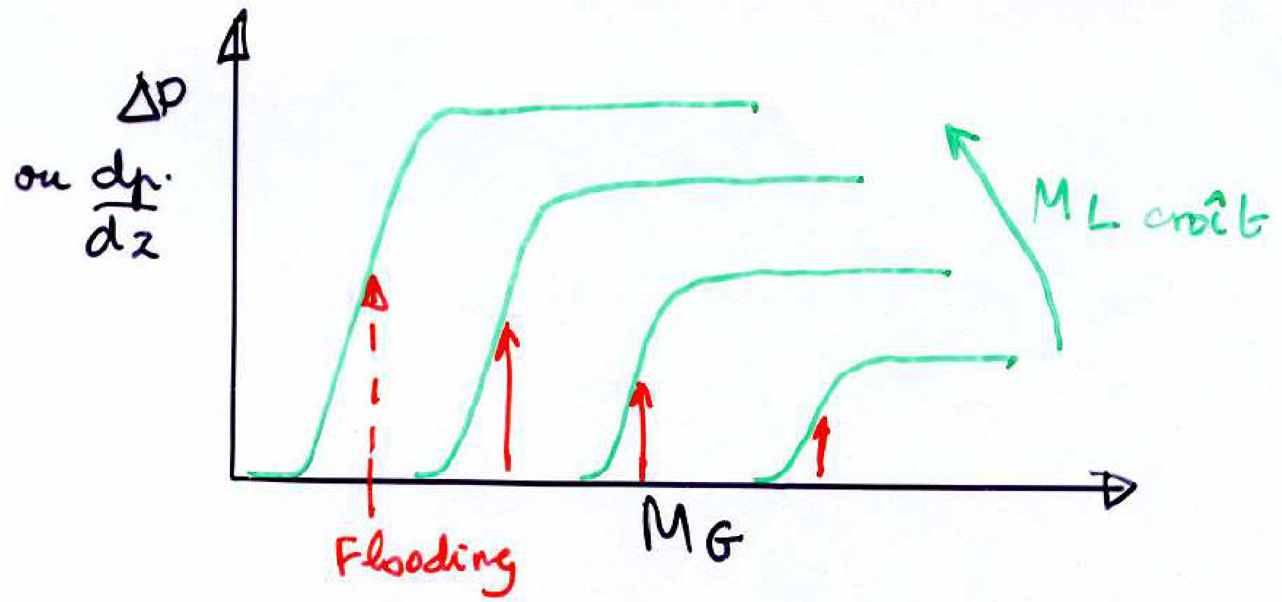
Air-water, 50 mm, 1 bar

FLOODING AND FLOW REVERSAL



Flooding: transition from counter-current towards co-current up flow. Flow reversal: reverse transition

EXPERIMENTAL DETERMINATION OF FLOODING



Modeling of flooding and flow reversal see also Bankoff & Chun Lee (1986).

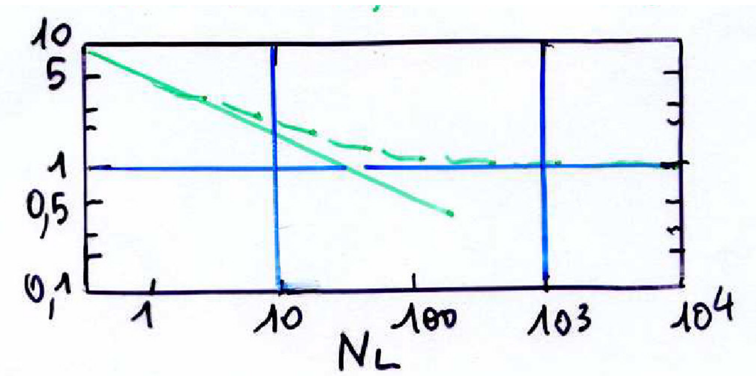
WALLIS MODEL

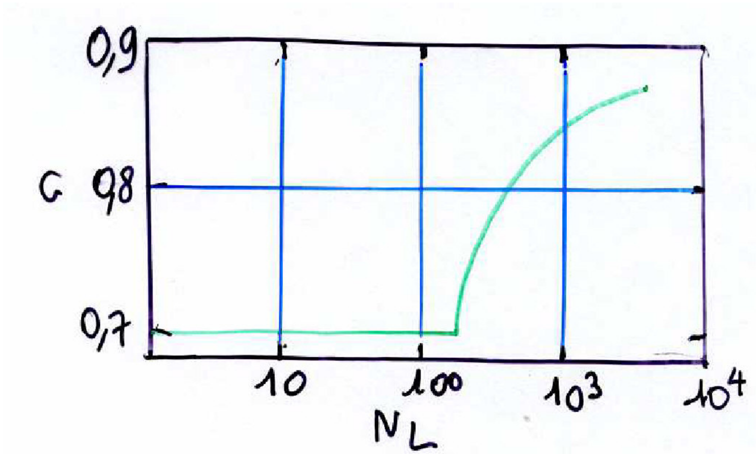
$J_k^* \approx$ Froude number.

$$J_G^* = \frac{J_G \rho_G^{\frac{1}{2}}}{(gD(\rho_L - \rho_G))^{\frac{1}{2}}}, \quad J_L^* = \frac{J_L \rho_L^{\frac{1}{2}}}{(gD(\rho_L - \rho_G))^{\frac{1}{2}}}, \quad \boxed{{J_G^*}^{\frac{1}{2}} + m {J_L^*}^{\frac{1}{2}} = C}$$

m and C depend on $N_L = \left(\frac{\rho_L g D^3 (\rho_L - \rho_G)}{\mu_L^2}\right)^{\frac{1}{2}} \equiv Gr$

$$N_L > 1000 \begin{cases} m = 1 \\ 0,88 < C < 1 \text{ (smooth inlet)} \\ C = 0,725 \text{ (sharp inlet)} \end{cases}, \quad N_L < 1000 \begin{cases} m = 5,6 N_L^{-1/2} \\ C = 0,725 \end{cases}$$





FLOODING ET FLOW REVERSAL

- Wallis model: no pipe length effect.
  - Some experiments show J_G^* increase with the increase of L. Favors the wave instability mechanism.
  - Many specific correlations.
- Flow reversal
  - Wallis model, $J_G^*(\mathrm{FR}) \neq J_G^*(\mathrm{Flooding})$, hysteresis effect, pipe diameter effect.

$$J_G^* = \frac{J_G \rho_G^{\frac{1}{2}}}{(gd(\rho_L - \rho_G))^{\frac{1}{2}}} = 0,5$$

  - Puskina and Sorokin model,

$$Ku = \frac{J_G \rho_G^{\frac{1}{2}}}{(g\sigma(\rho_L - \rho_G))^{\frac{1}{4}}} = 3,2$$

- Control mechanisms: still an open problem.

REFERENCES

Bankoff, S. G., & Chun Lee, Sang. 1986. *Multiphase Science and Technolgy*. Vol. 2. Hemisphere. Chap. 2-A critical review of the flooding literature, pages 95–180.

Delhaye, J.-M. 2008. *Thermohydraulique des réacteurs nucléaires*. Collection génie atomique. EDP Sciences.

Dukler, A. E., & Taitel, Y. 1986. *Multiphase Science and Technolgy*. Vol. 2. Hemisphere. Chap. 1-Flow pattern transitions in gas-liquid systems: measurement and modelling, pages 1–94.