A SHORT INTRODUCTION TO TWO-PHASE FLOWS

Void fraction: Experimental techniques and simple models

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PHASE PRESENCE FUNCTION

- Phase presence function definition:

$$X_k(\mathbf{r}, t) \triangleq \begin{cases} 1 & \text{si } M(r) \in \text{ phase } k, \\ 0 & \text{si } M(r) \notin \text{ phase } k. \end{cases}$$

- Phase indicator, *FIP, fonction indicatrice de phase*, χ_k.
- Space averaging:

$$\text{Conditional, } < f >_n \triangleq \frac{1}{D_{kn}} \int_{D_{kn}} f_k \mathrm{d}V,$$

$$\text{Plain, } \not< f \not>_n \triangleq \frac{1}{D_n} \int_{D_n} f \mathrm{d}V.$$

AVERAGING OPERATORS (CT'D)

- Time averaging:

$$\text{Conditional, } \overline{f}_k^X(t) \triangleq \frac{1}{T_k} \int_{[T_k]} f(\tau) \mathrm{d}\tau,$$

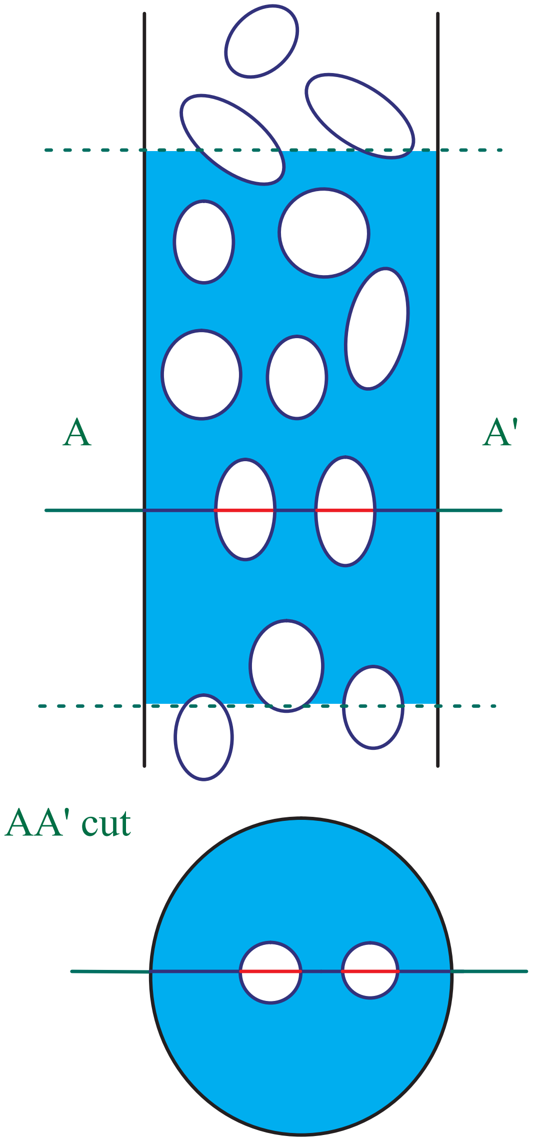
$$\text{Plain, } \overline{f}(t) \triangleq \frac{1}{T} \int_{[T]} f(\tau) \mathrm{d}\tau.$$

- Commutativity of averaging operators:

$$\overline{R_{kn} < f_k >_n} = \nless \alpha_k \overline{f}_k^X \ngtr_n .$$

- Void fraction: average of the phase presence function
- Void fraction, gas hold-up: *taux de présence du gaz*, *taux de vide*.

VOID FRACTION (α)



- Local time fraction (gas, void fraction) :

$$\alpha_G(\mathbf{r},t) \triangleq \overline{X_G} = \frac{T_G}{T}.$$

- Instantaneous space fraction.
  - Line fraction:

$$R_{G1}(t) \triangleq \langle X_G \rangle_1 = \frac{L_G}{L_G + L_L} = \frac{L_G}{L}$$

  - Area fraction:

$$R_{G2}(t) \triangleq \langle X_G \rangle_2 = \frac{A_G}{A_G + A_L} = \frac{A_G}{A}$$

  - Volume fraction:

$$R_{G3}(t) \triangleq \langle X_G \rangle_3 = \frac{V_G}{V_G + V_L} = \frac{V_G}{V}$$

BASIC IDENTITIES

- Commutativity ($f = 1$): $\overline{R_{kn}} = \langle\!\langle \alpha_k \rangle\!\rangle_n \quad = \overline{\langle\!\langle X_k \rangle\!\rangle_n}$.
- Mean phase fraction.
  - Mean line-averaged,

$$\overline{R_{G1}} = \frac{1}{T} \int_{[T]} R_{G1}(\tau)\,\mathrm{d}\tau = \frac{1}{L} \int_L \alpha_G \mathrm{d}L$$

  - Mean area-averaged,

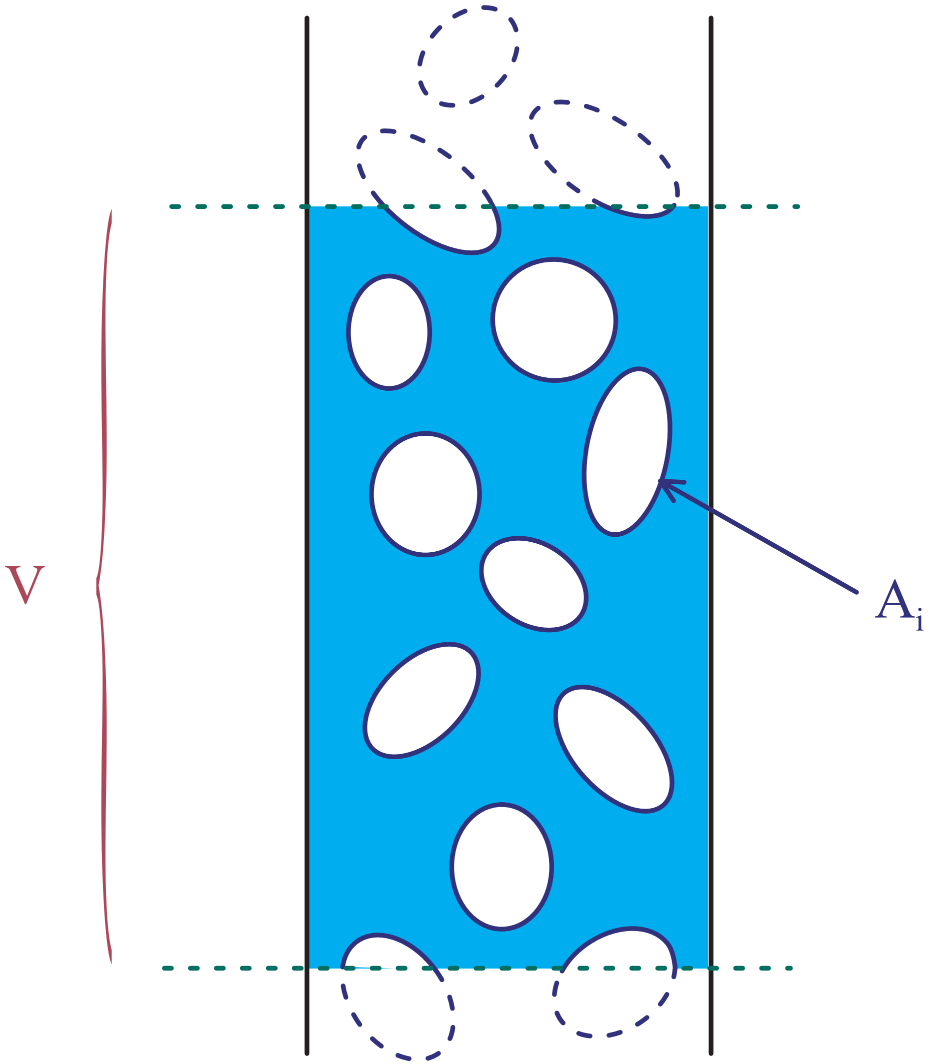
$$\overline{R_{G2}} = \frac{1}{T} \int_{[T]} R_{G2}(\tau)\,\mathrm{d}\tau = \frac{1}{A} \int_A \alpha_G \mathrm{d}A$$

  - Mean volume averaged,

$$\overline{R_{G3}} = \frac{1}{T} \int_{[T]} R_{G3}(\tau)\,\mathrm{d}\tau = \frac{1}{V} \int_V \alpha_G \mathrm{d}V$$

- 7 precise definitions of the void fraction. The one to keep depends on the context (model). VF is always some average of X_k.

OTHER RELATED DEFINITIONS



- Instantaneous interfacial area:

$$\Gamma_3(t) \triangleq \frac{A_i(t)}{V}$$

- Local interfacial area, is a time-averaged quantity:

$$\gamma = \sum_{\text{disc.}\in[T]} \frac{1}{|\mathbf{v}_i.\mathbf{n}_k|}$$

- Identity (commutativity of interaction terms), mean interfacial area:

$$\overline{\Gamma_3} \equiv \nleftarrow \gamma \nrightarrow_3$$

- Γ_3 and γ can be measured.

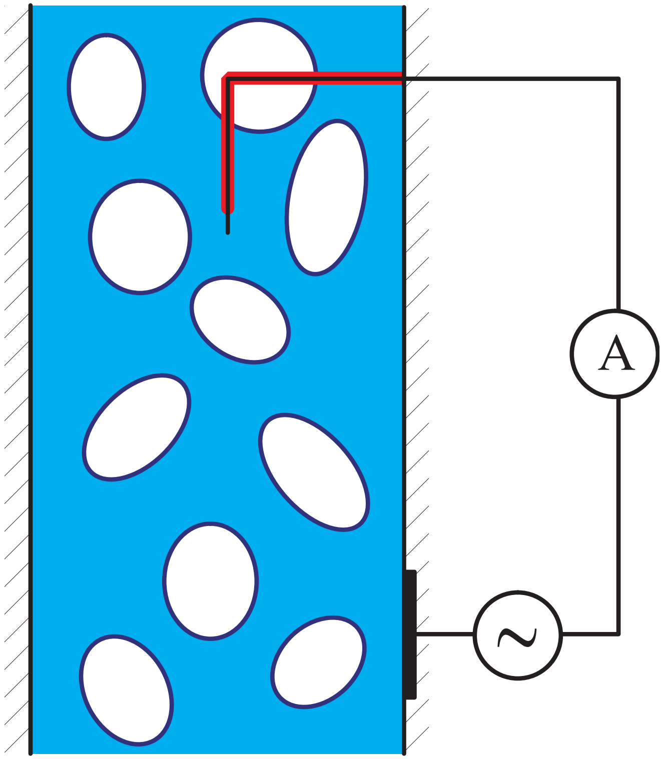
EXPERIMENTAL TECHNIQUES FOR VOID FRACTION

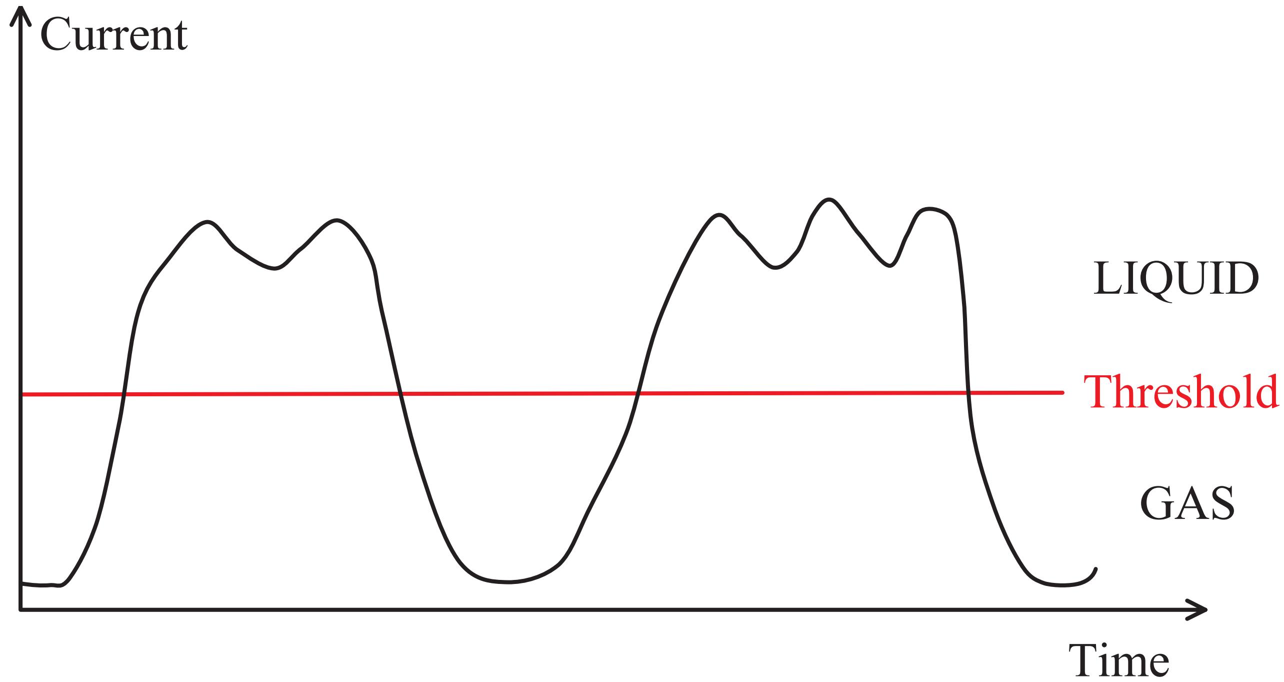
- Local void fraction.
  - Electrical probes
  - Optical probes
- Line averaged void fraction.
  - Light attenuation (X or γ rays)
- Area-averaged void fraction.
  - X-rays ou γ-rays, (one-shot)
  - Multi-beam densitometry
  - Neutrons diffusion (steel, steam-water, HP-HT)
  - Impedance probes
- Volume averaged void fraction,
  - Quick closing valves,
  - Gravitational (hydrostatic) pressure drop,
  - Ultrasound attenuation (Bensler, 1990).
- Medical imaging, CT, MRI.

LOCAL VOID FRACTION

Electrical probes (different resistivity):
Determination of the liquid phase indicator, $X_L(\mathbf{r}, t)$.

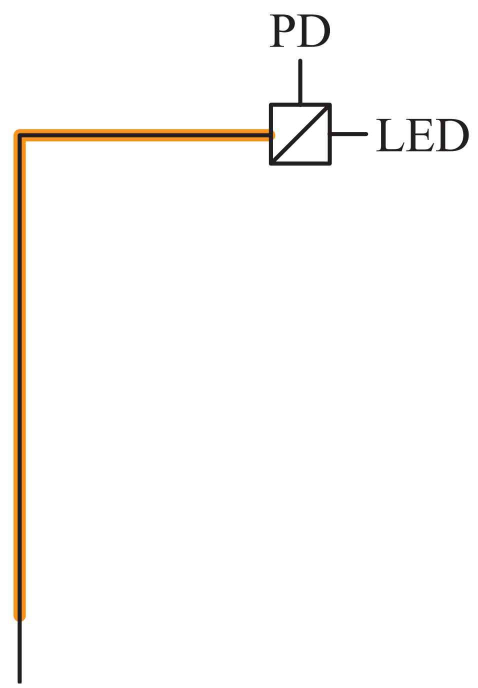
- Continuous phase (water), conducting,
- Dispersed phase (air), non conducting
- Threshold on current $\rightarrow X_L \rightarrow \alpha_L$



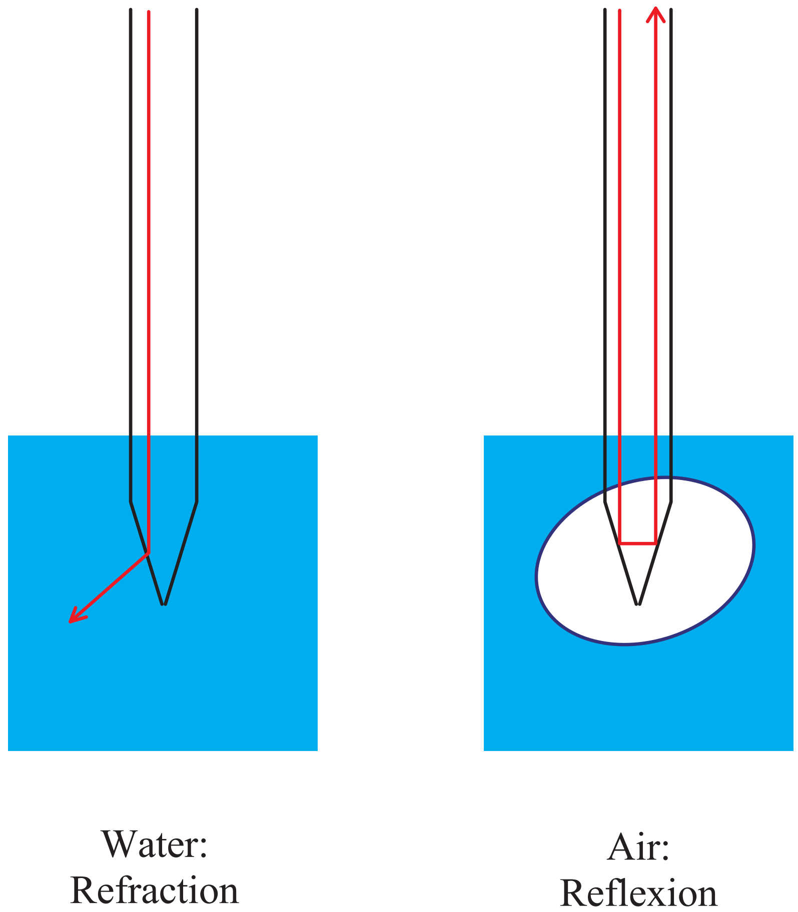


LOCAL VOID FRACTION

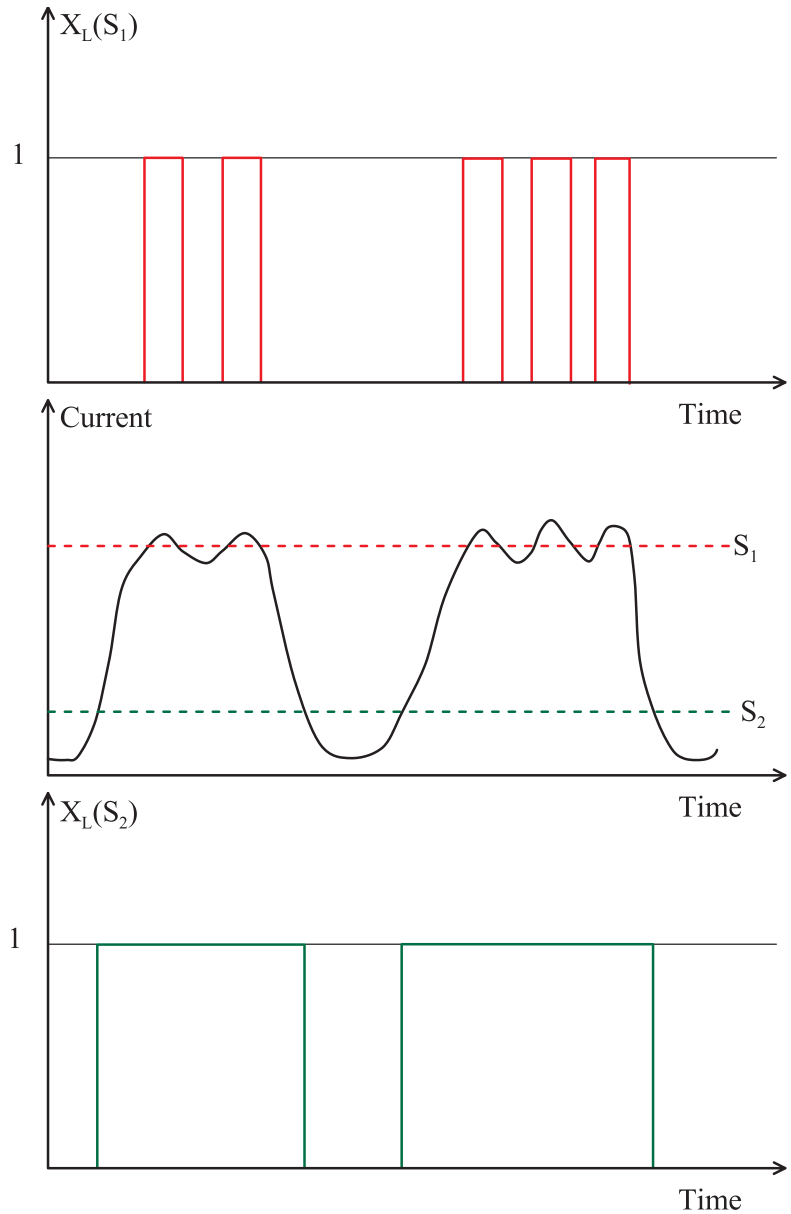
Optical probes (different refraction index) :
Determination of the gas indicator, $X_G(\mathbf{r}, t)$.
Principle: dish washer, rinsing liquid level indicator.



Water, freons, $T < 110°\mathrm{C}$



HOW TO SET THE THRESHOLD LEVEL?



- $\alpha = \overline{X_L}$, depends on threshold:

$$S_1 > S_2 \Rightarrow \alpha_{L1} < \alpha_{L2}.$$

- Reference method:

$$\Delta p \rightarrow \overline{R_{G2}}$$

- It is recalled,

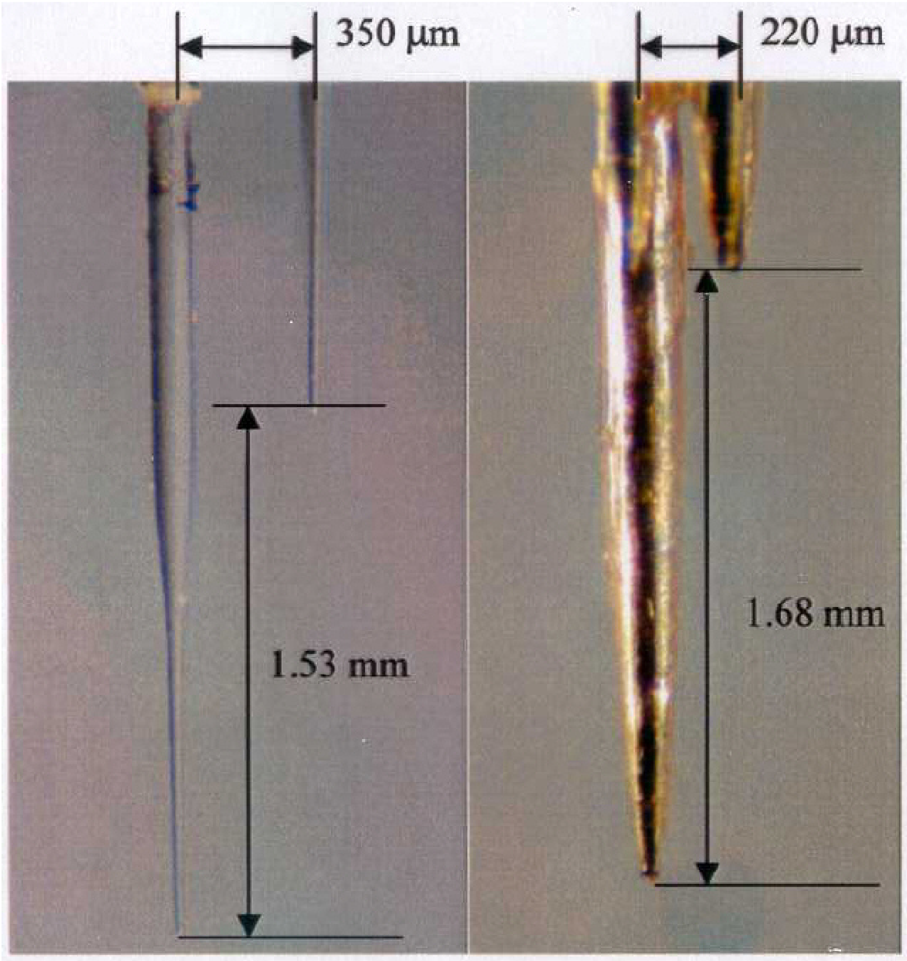
$$\nleftarrow \alpha_G \nrightarrow_2 = \overline{R_{G2}}$$

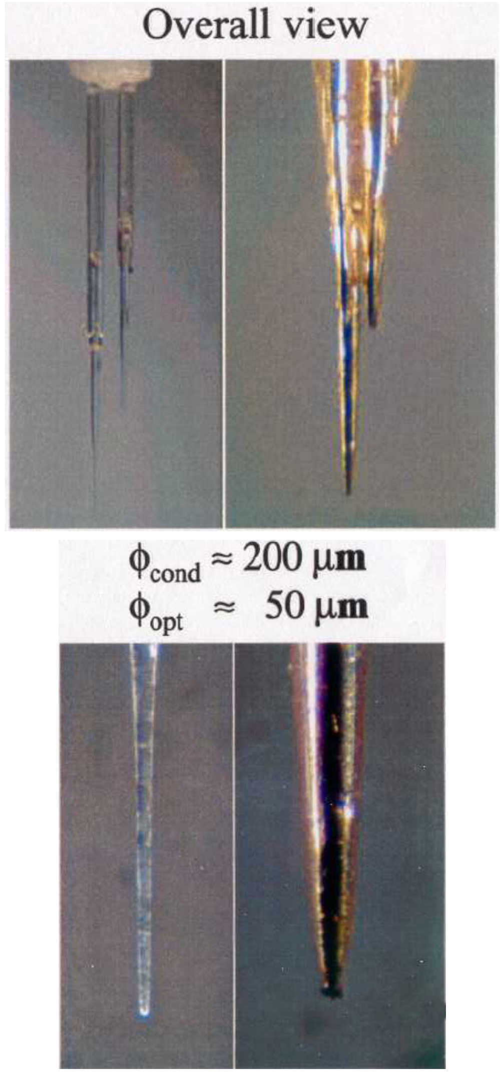
- Determine $\alpha_G(S)$ on the section. Find S such that,

$$\nleftarrow \alpha_G(S) \nrightarrow_2 = \overline{R_{G2}}$$

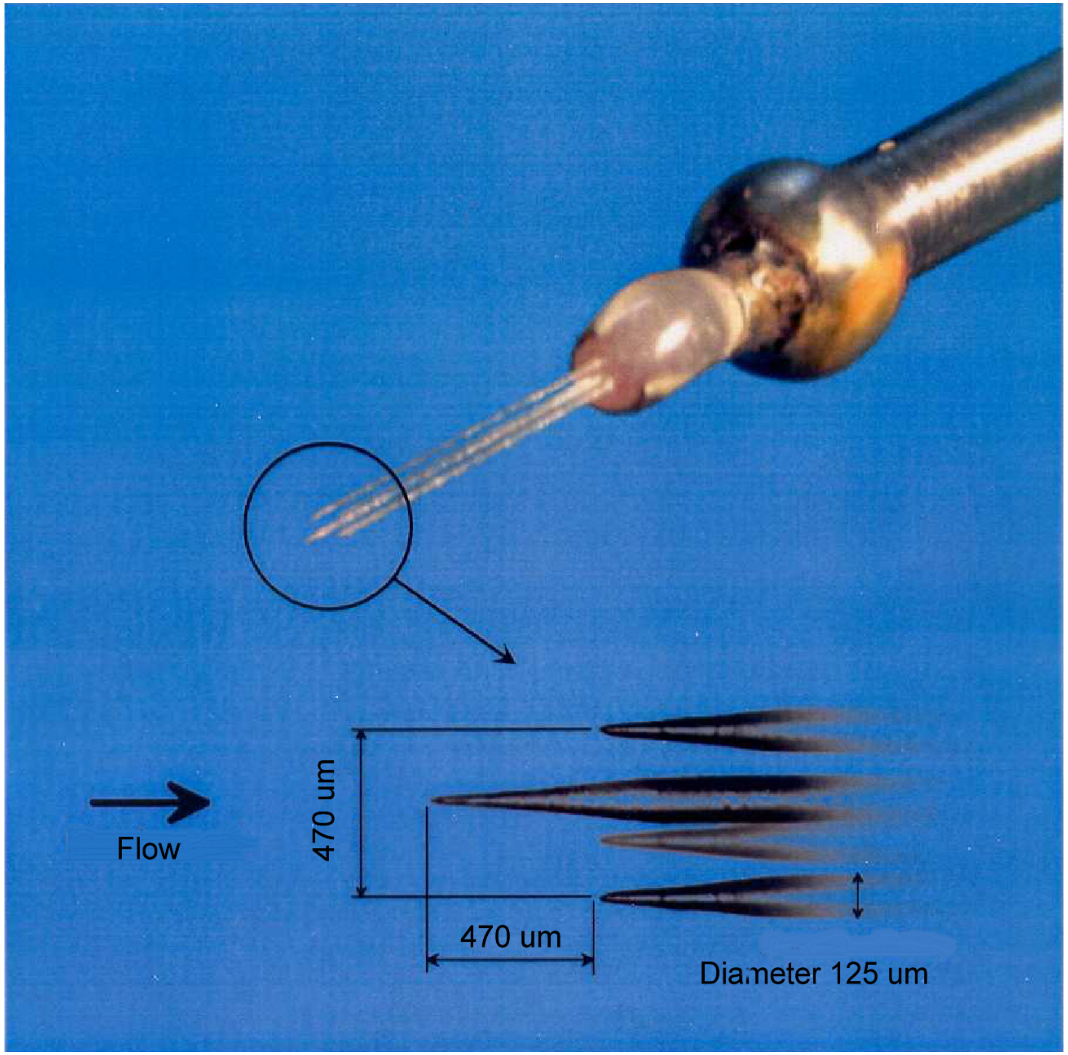
- Consistency check, not a calibration.

ELECTRICAL & OPTICAL PROBES





MULTIPLE TIP SENSORS



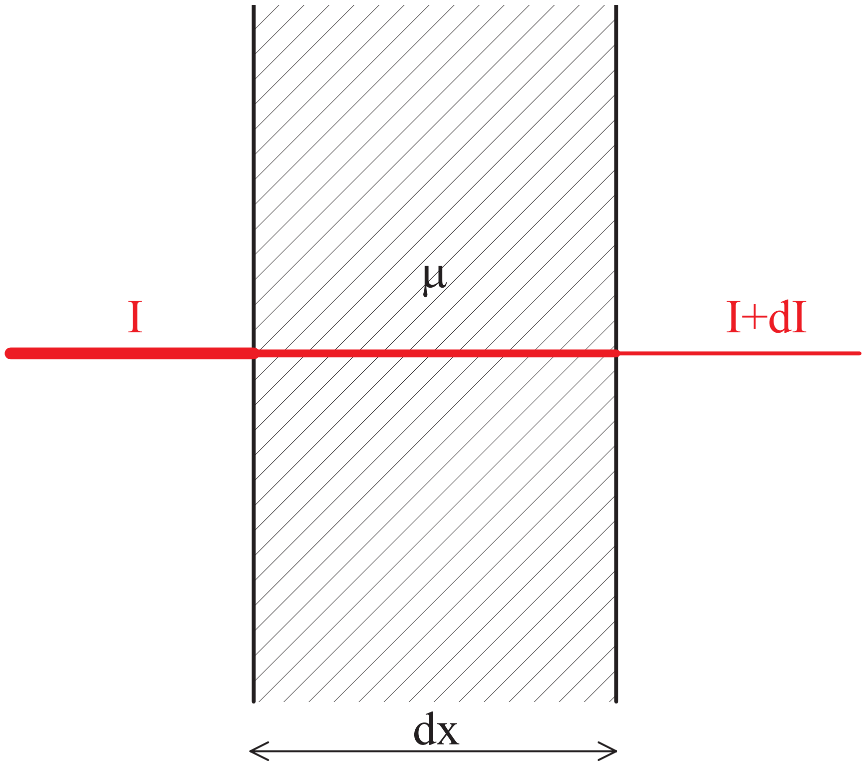
- 2-sensor probe. Add assumptions: spherical bubbles, chord distribution→mean diameter, mean gas velocity.
- 4-sensor probe. Determine interface orientation ($\mathbf{n}_k$), geometrical surface velocity, $\mathbf{v}_i \centerdot \mathbf{n}_k$
- Local interfacial area,

$$\gamma = \sum_{\text{disc.}\in[\text{T}]} \frac{1}{|\mathbf{v}_i.\mathbf{n}_k|}$$

- Mean sauter diameter (D_{32}), identity (bubbles),

$$\gamma \equiv \frac{6\alpha}{D_{SM}}$$

LIGHT (PHOTONS) ATTENUATION

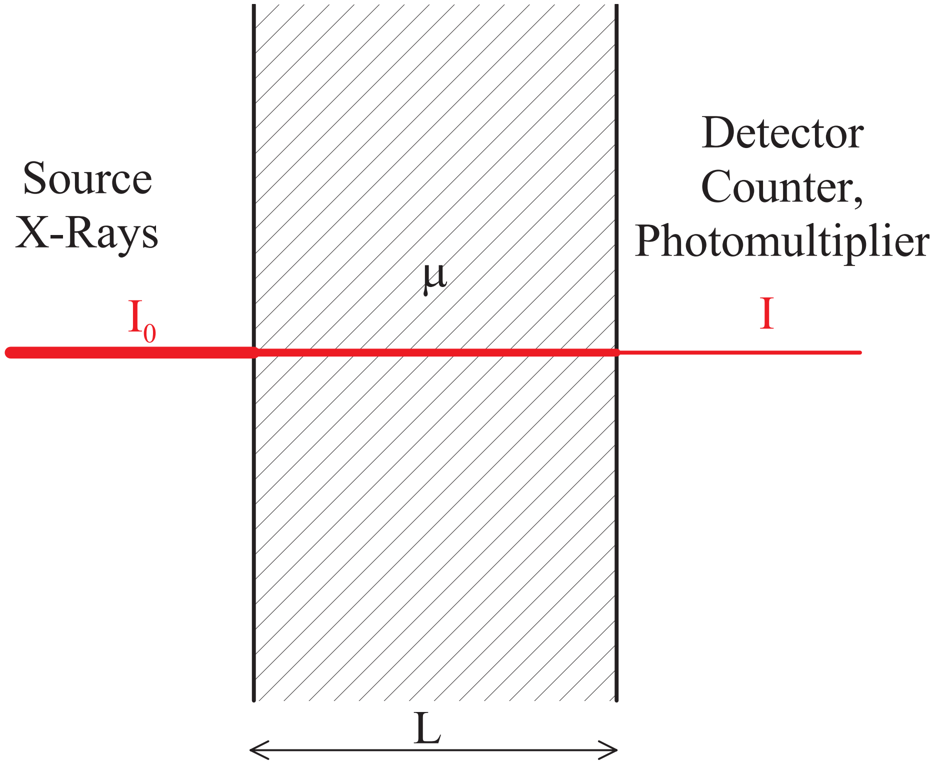


- X-rays or γ-rays.
- Collimated beam, single spectral line
- Beer-Lambert relation:

$$\mathrm{d}I = -\mu I \mathrm{d}x, \quad [\mu] = \mathrm{L}^{-1}$$

- Exponential absorption, μ absorption coefficient.
- $\dfrac{\mu}{\rho}$: mass energy-absorption coefficient depends on f.

PRACTICAL IMPLEMENTATION

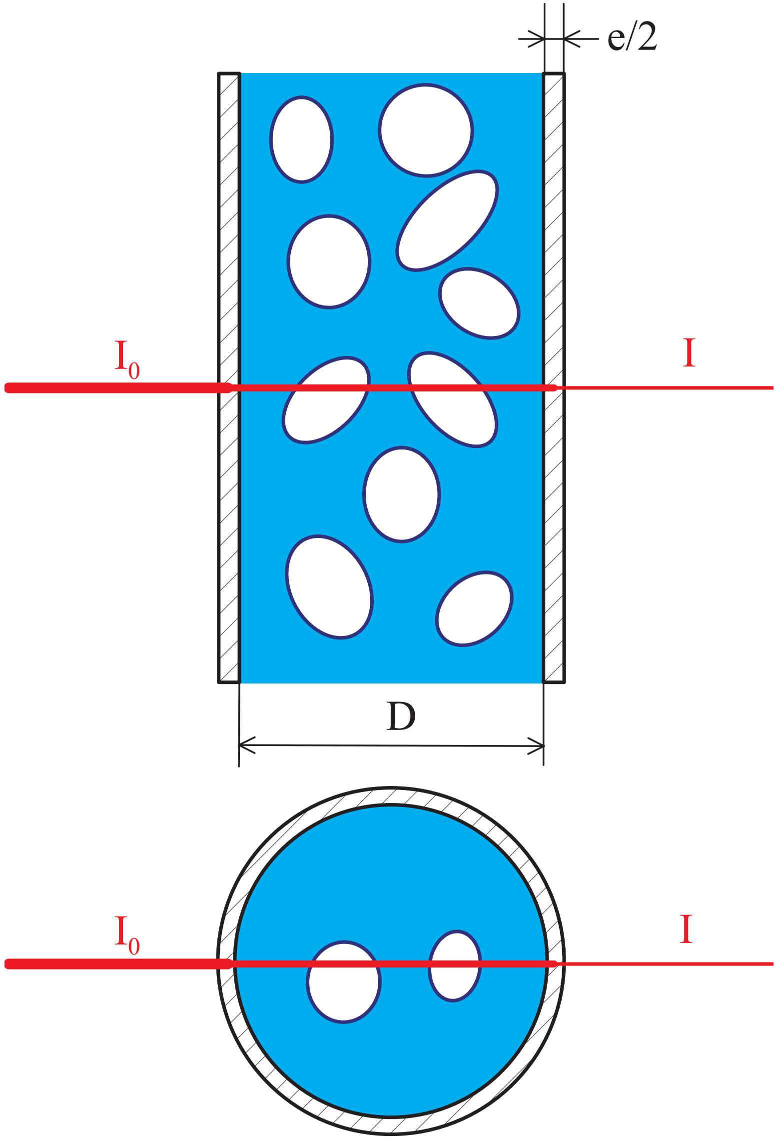


- X-Rays generator, γ source
- Detection: photo multiplier (NaI, semi-conductors), counter.
- Collimation: thick heavy metal block, drilled, 0,5 mm
- Collimated beam, single spectral line,
- Integrate B-L on a finite length,

$$I = I_0 \exp(-\mu L) = I_0 \exp\left(-\frac{\mu}{\rho}\rho L\right)$$

- At low pressure, little absorption in the gas.

MEASURING THE LINE PHASE FRACTION



Air-water flow, steam-vapor.

- D, diameter, $e/2$ wall thickness.
- Beer-Lambert equation:

$$I = I_0 \exp(-\mu_p e) \exp\left(-\mu_L (1 - R_{G1}) D\right) \exp(-\mu_G R_{G1} D)$$

- Definition of the gas line fraction:

$$R_{G1}(z,t) \triangleq \frac{L_G}{L_G + L_L} = \frac{L_G}{D}$$

- Low pressure assumption:

$$I_G = I_0 \exp(-\mu_p e)$$
$$I_L = I_0 \exp(-\mu_p e) \exp\left(-\mu_L D\right)$$
$$I = I_0 \exp(-\mu_p e) \exp\left(-\mu_L (1 - R_{G1}) D\right)$$

$$\boxed{R_{G1} = \frac{\ln I/I_L}{\ln I_G/I_L}}$$

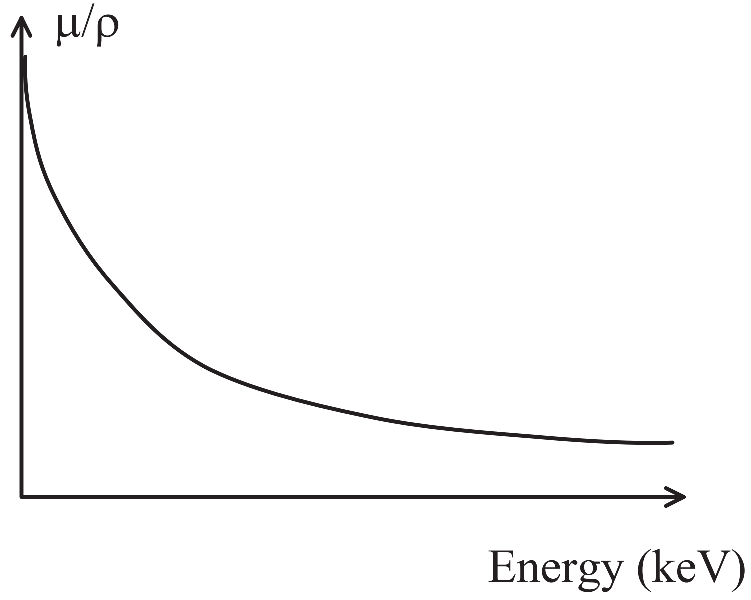
UNCERTAINTY FACTORS

- Contrast → low energy

$$\frac{I_G}{I_L} \approx \exp\left(\frac{\mu_L}{\rho_L}\rho_L D\right)$$

- Statistical errors with counters → high energy

$$I \propto N, \quad \frac{\Delta N}{N} \propto \sqrt{\frac{1}{N}}$$

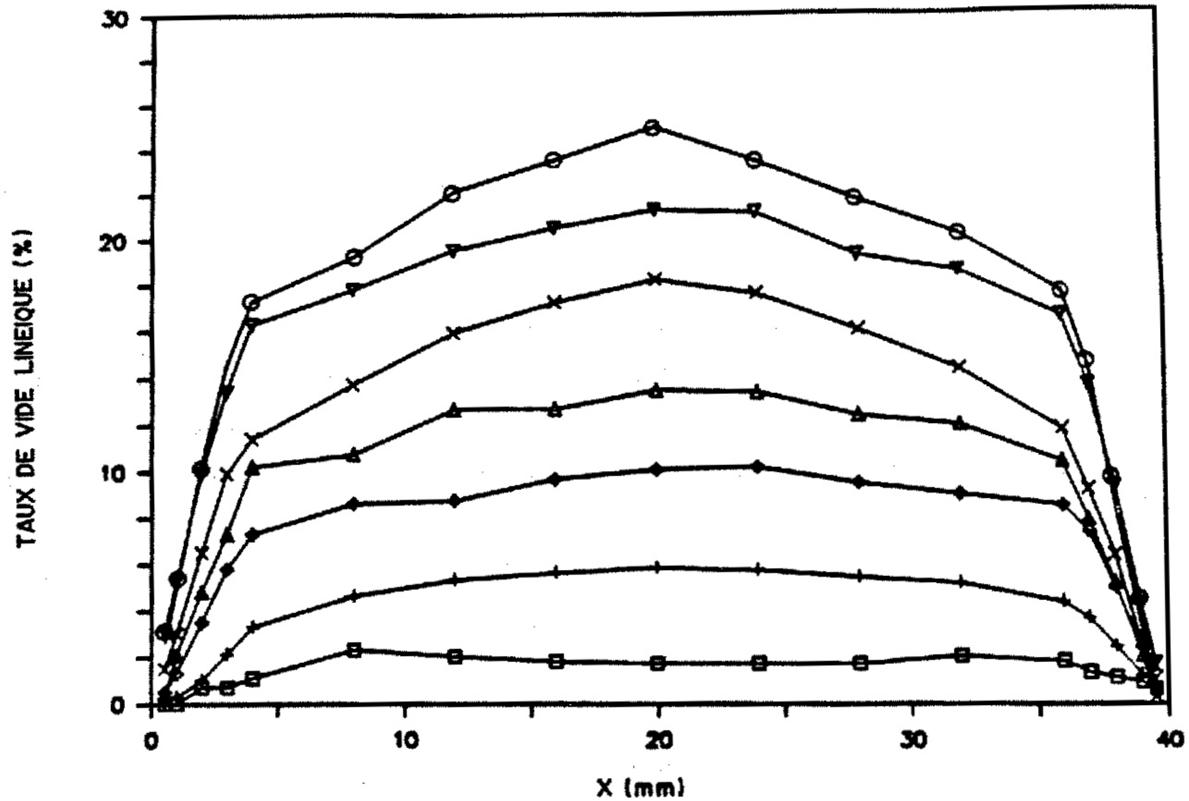


- R_{L1} fluctuations, $I \propto \exp(R_{G1})$ and $\overline{\exp f} \neq \exp \overline{f}$,

$$\Delta R_G \approx 0,20 \text{ (slug)}, \quad \Delta R_G \approx 0,05 \text{ (churn)}.$$

- Source stability: reference beam method, $I \to \frac{I}{I_0'}$.
- Spectral hardening, direct calibration, $I(R_L)$, or use filters.

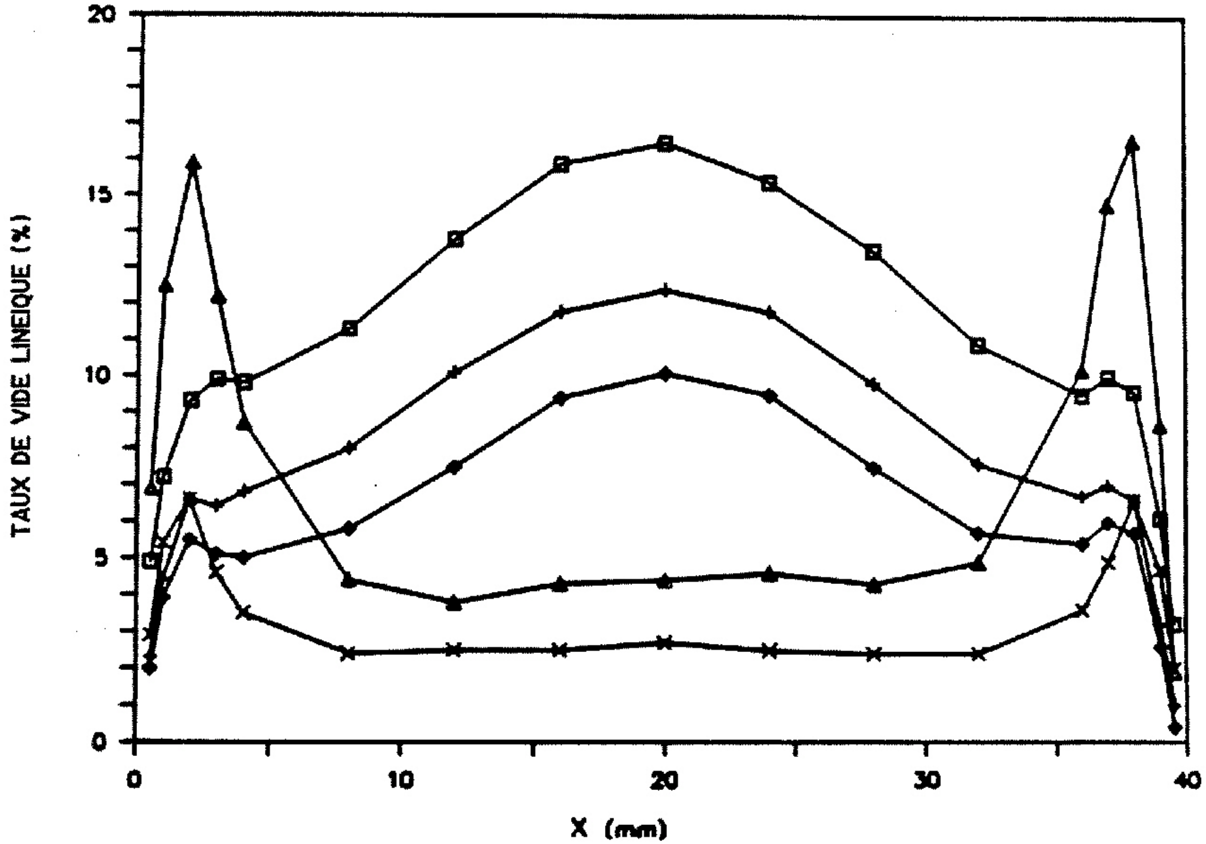
MEAN LINE GAS FRACTION



After Bensler (1990, p. 60)

- No water flow, $J_L = 0$: $\overline{R_{G2}} = 0{,}01, 0{,}04, 0{,}07, 0{,}10, 0{,}13, 0{,}16, 0{,}19$.

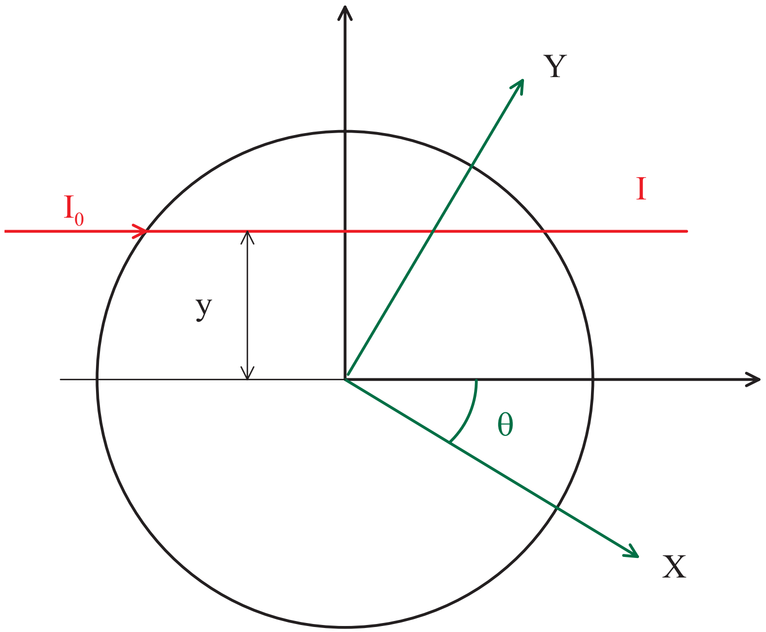
MEAN LINE GAS FRACTION



After Bensler (1990, p. 61)

- Two-phase flow, $J_L = 2$ m/s : $\overline{R_{G2}} = 0{,}03,\ 0{,}061,\ 0{,}069,\ 0{,}089,\ 0{,}123.$
- Wall peaking, still an open problem for modeling...
- Transition in profile shape, bubble clustering, slug flow.

MEAN AREA FRACTION



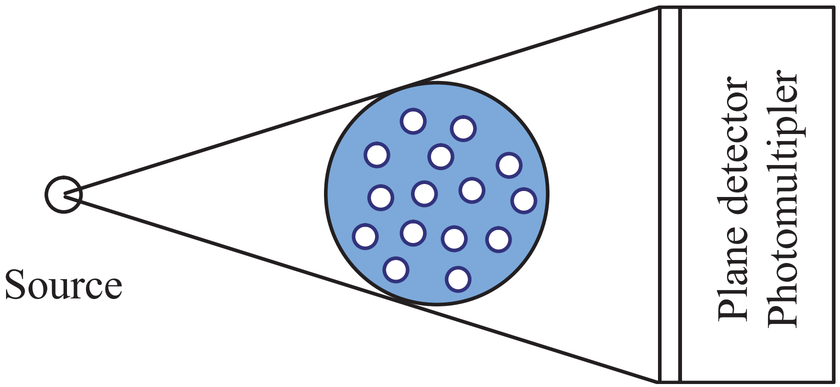
- Mean area fraction, $\overline{R_{G2}}$,

$$\overline{R_{G2}} = \frac{1}{\pi R^2} \int_{-R}^{R} \overline{R_{G1}}(y) \sqrt{R^2 - y^2} \mathrm{d}y$$

- Computed tomography, axis-symmetric,

$$\overline{R_{G1}}(y) \Leftrightarrow \alpha_G(r)$$

$$\overline{R_{G1}}(y, \theta) \Leftrightarrow \alpha_G(X, Y)$$

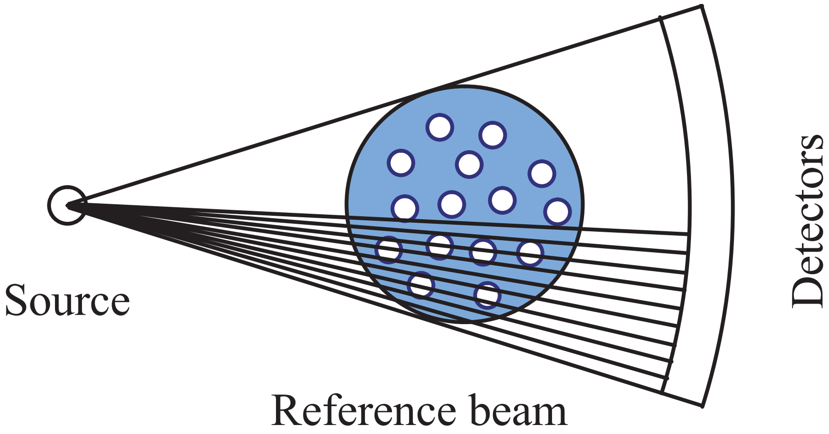


- Instantaneous surface fraction, $R_{G2}(t)$
- Known limitations, Compton, diffusion

$$\Delta R_{G2} \leqslant 0,05$$

$$0 < R_{G2} < 0,8$$

MEAN AREA FRACTION



- Multibeam densitometer,

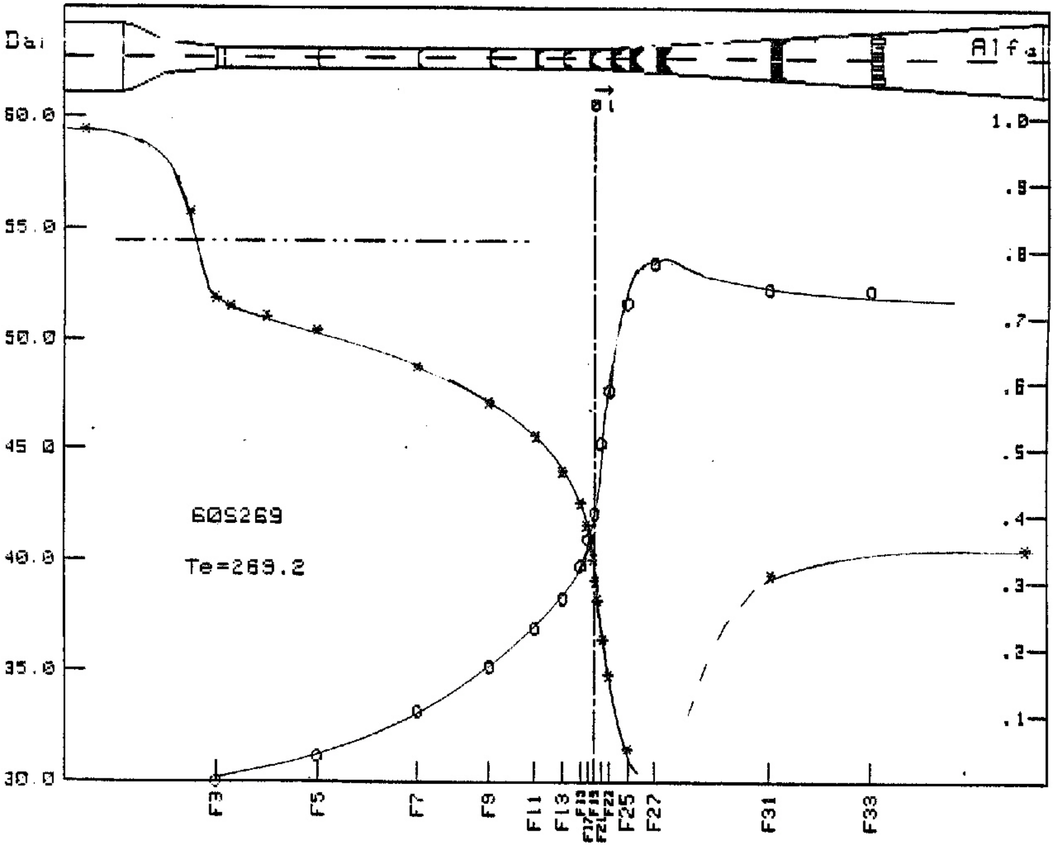
$$R_{G1}(\theta) \Leftrightarrow \alpha_G(r)$$

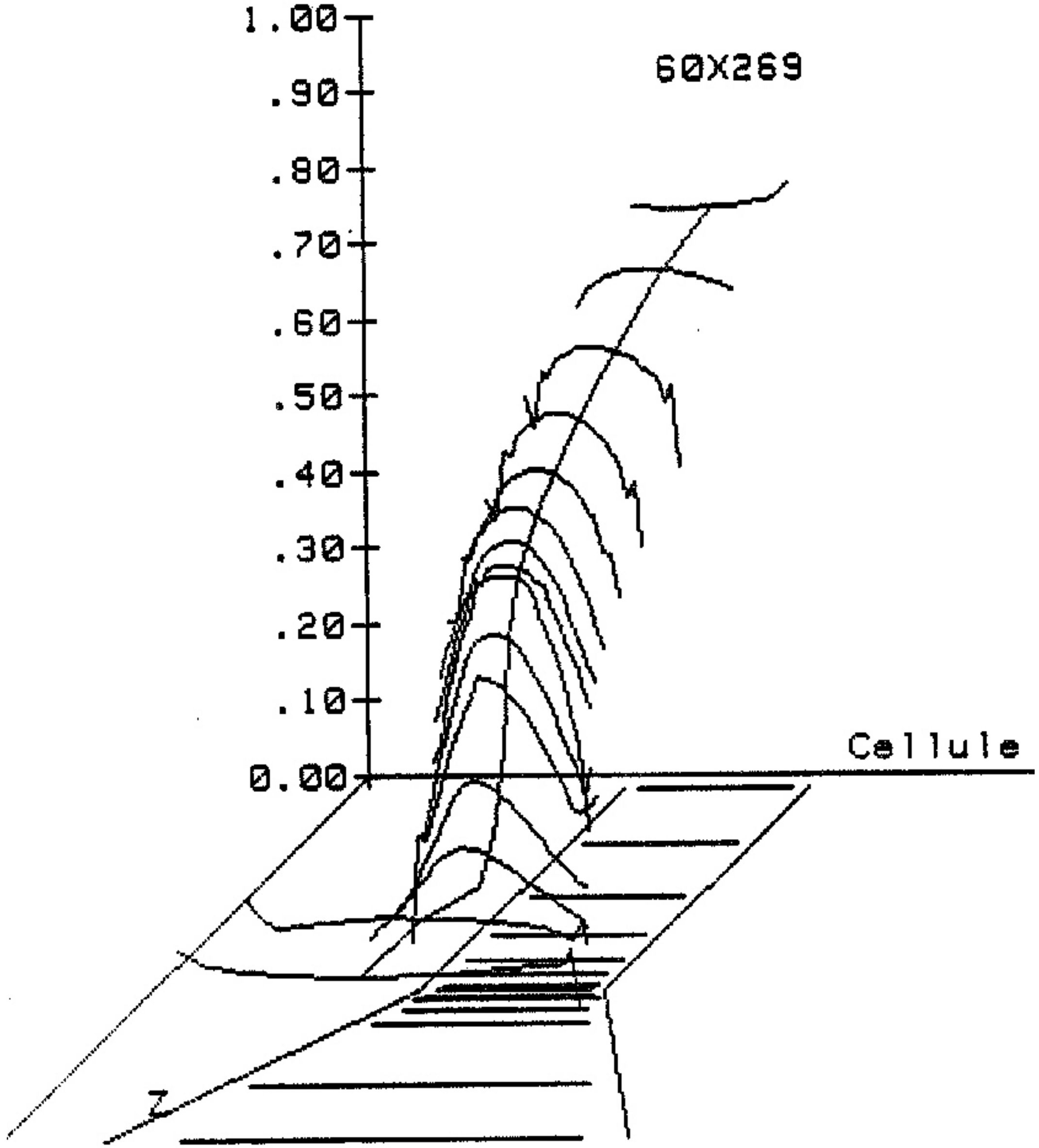
- XCT, medical scanner

$$\overline{R_{G1}}(\theta,\phi) \Leftrightarrow \alpha_G(x,y)$$

- Neutron diffusion, 90 $^\circ$
- Low attenuation in steel, diffusion on H nucleus
- Neutron cinematography.

SUPER MOBY DICK



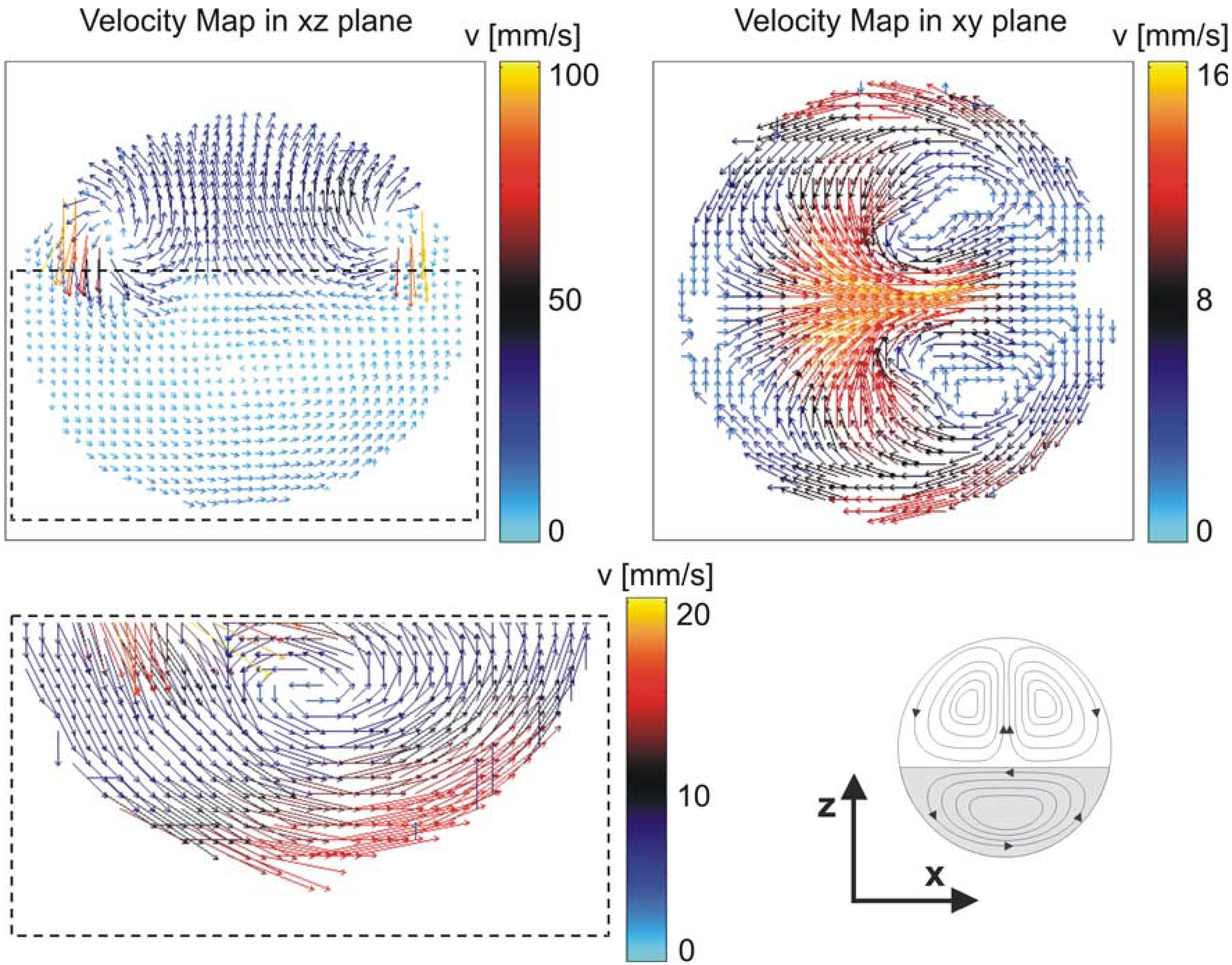


After figures 63'r et 64'r
from Jeandey *et al.* (1981).
$P_0 = 59.43$ bar, $T_L = 269.2$ °C,
$P_{\mathrm{sat}} = 54.31$ bar.

THE ULTIMATE METHOD?

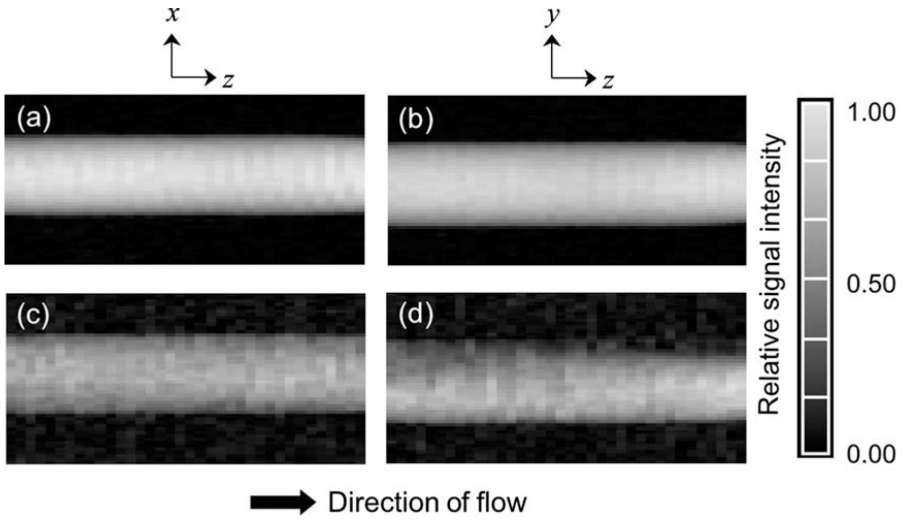
- Nuclear magnetic resonance, NMR, MR imaging
  - Non intrusive method,
  - Magnetization (H, F), magnetic fields
  - Density (void fraction), velocity
- Space and time resolution
  - 0D, 1D, 2D, etc.
  - Time averaged quantities, arbitrary space filters (LES).
  - Turbulent transport phenomena.
- Routine for medicine, body (static) imaging, 1 mm^3, arterial flow rate, still under development for flow imaging.

FLOW IMAGING OF A LEVITATED DROP

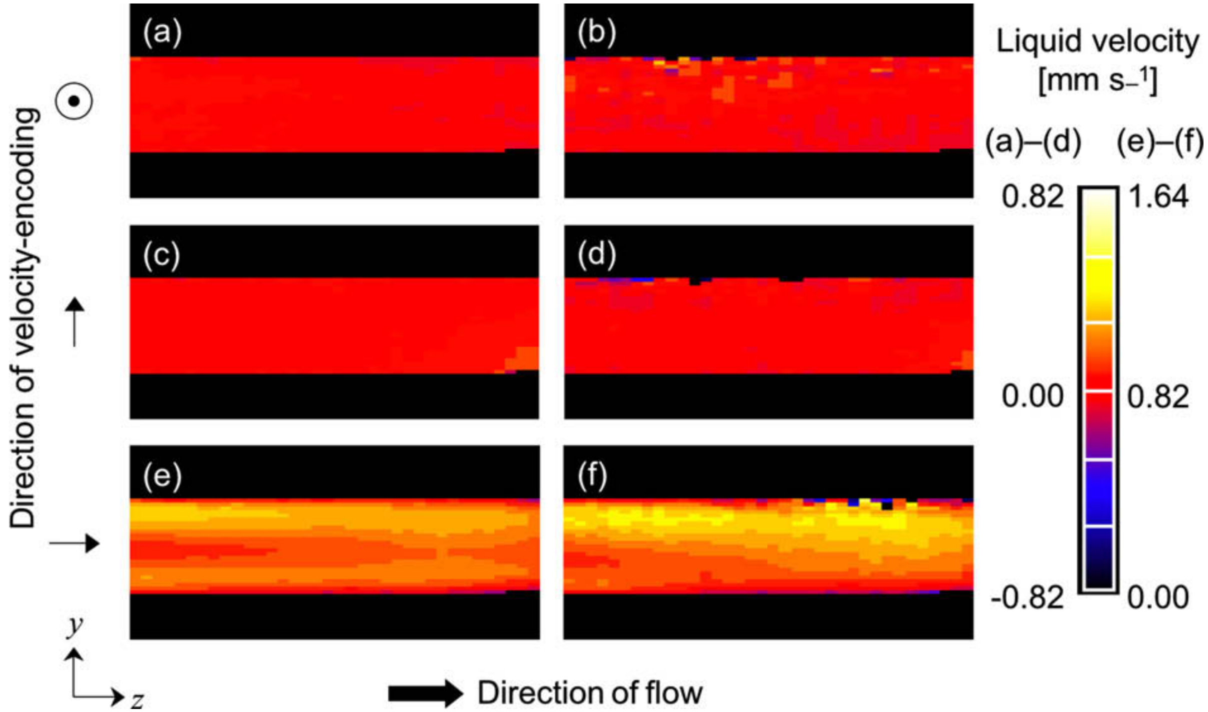


Velocity imaging, after Amar *et al.* (2005, Fig. 13).

VELOCITY & VOID FRACTION IN BUBBLY FLOW



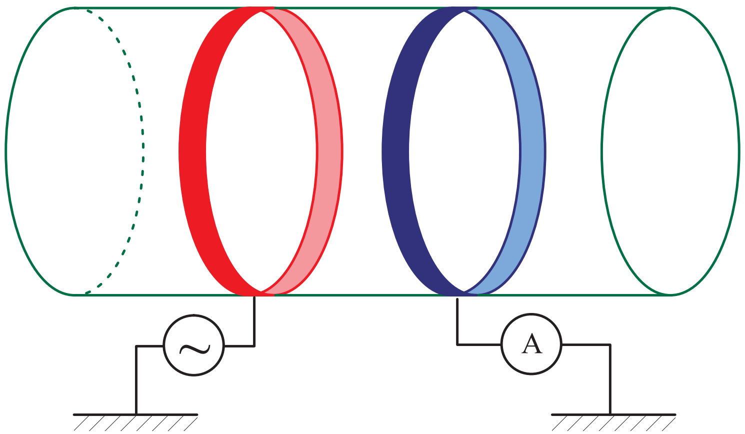
Liquid fraction, $\overline{R_{L1y}}$ and $\overline{R_{L1x}}$

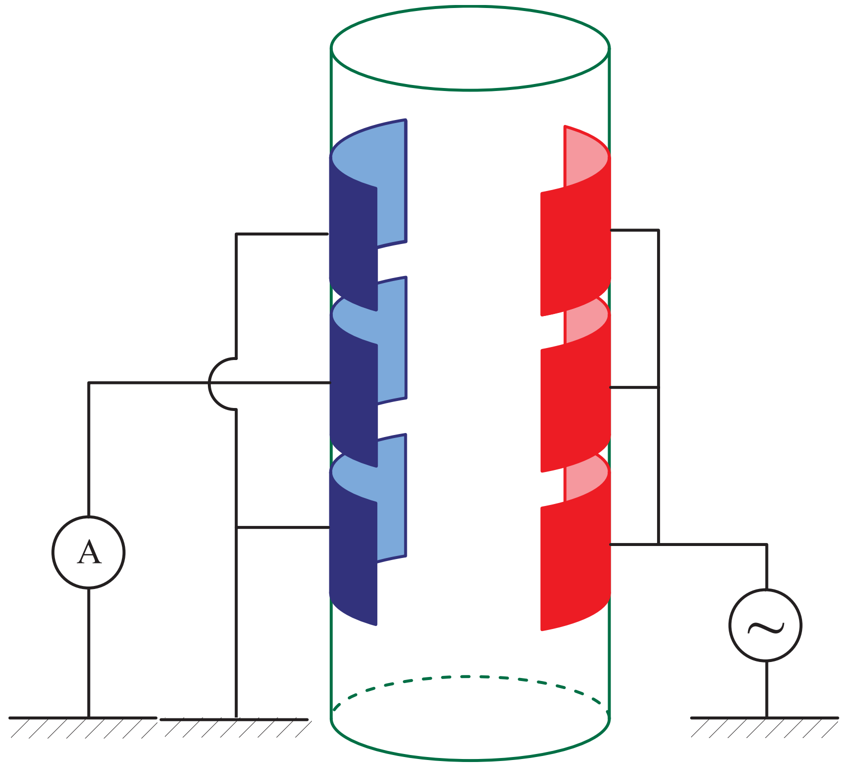


Mean 1D liquid velocity, $\lessdot \overline{v_{iL}}^{X_L} \gtrdot_{1x}$.

Horizontal bubbly flow, $D = 13.9$ mm, after Sankey *et al.* (2009, Figs 7 and 10). Velocity scale is m/s, not mm/s.

IMPEDANCE DENSITOMETRY





- Two-phase medium impedance, excitation voltage, E, signal: current I.

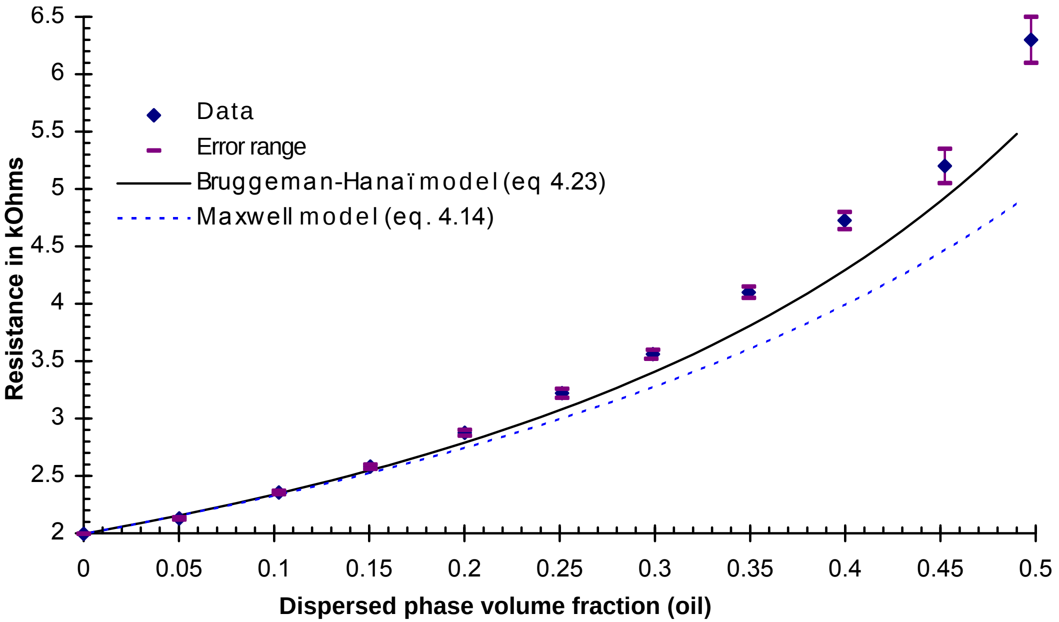
$$I = DE\sigma_C(T, c_1, c_2, \cdots)f(R_{G3}, \cdots)$$

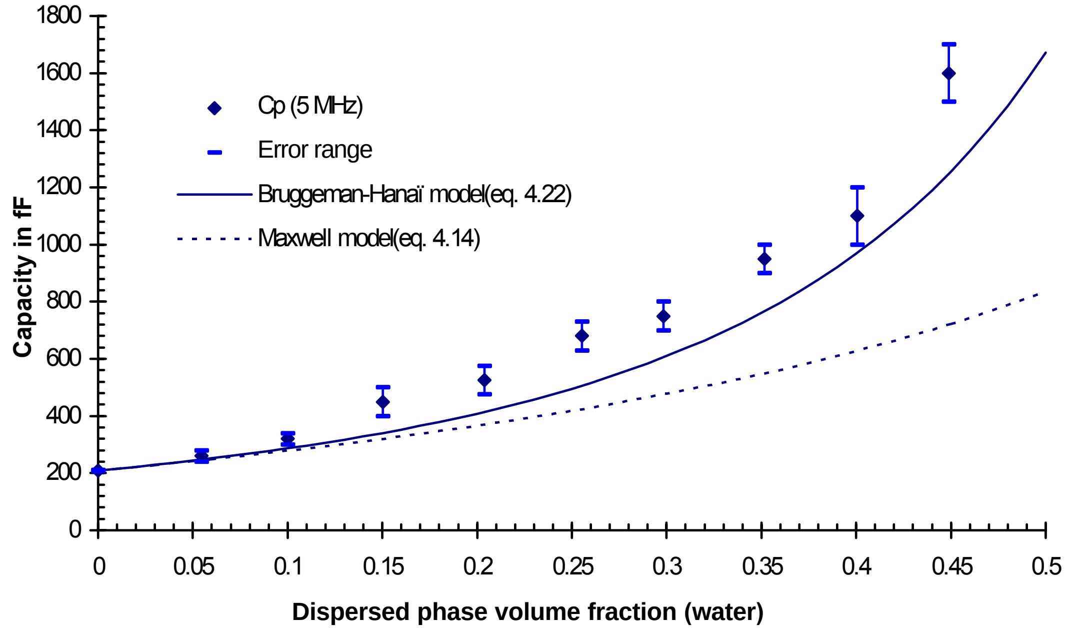
- Resistive, $\sigma_{2\phi}$, capacitive, $\epsilon_{2\phi}$
- Minimize effect of impedance electrode-medium, $10 < f < 100$ kHz

COMPOSITION-IMPEDANCE RELATION

- Electrode pattern: depends on flow regime.
  - Rings: stratified flow, quasi-linear $I(R_{L2})$, 1D-conductor.
  - Facing electrodes, confine the measuring volume (guard electrodes), bubbly flow, density waves.
- Small integration volume: $R_{G3} \approx R_{G2}(t)$
- Temperature sensitivity: 1°C$\approx$ 1% void fraction.
- Reference method, compensate for effects σ_C variations
  $I \rightarrow \frac{I}{I_0}$, $I_0 = DE\sigma_C(T, c_1, c_2, \cdots)f(0)$
- Calibration (reference method), numerical modeling (BEM)
- Optimization of electrodes shape (BEM): $f(R_{G2}, \cdots) \approx g(R_{G2})$.

OIL-WATER FLOWS



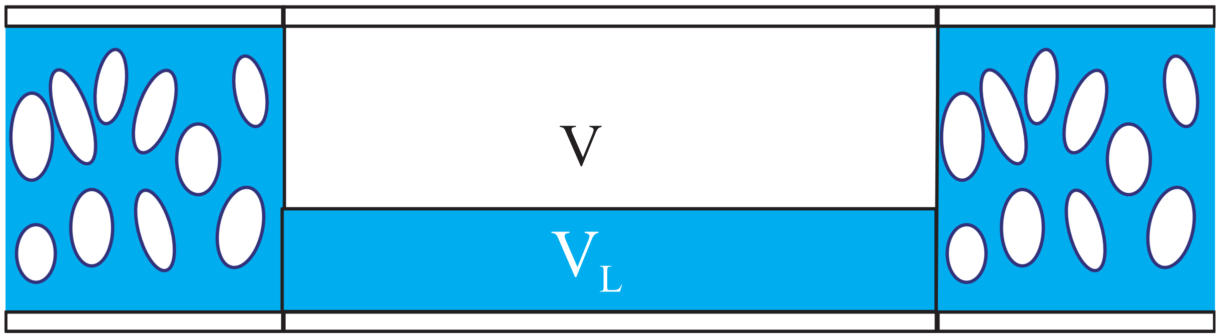


- After Boyer (1992, p. 98)
- Theory for dispersions, Maxwell, Bruggemann, $\sigma_D/\sigma_C \to 0$,

$$\sigma_{2\phi} \approx \sigma_C (1 - R_{D3})^{3/2}$$

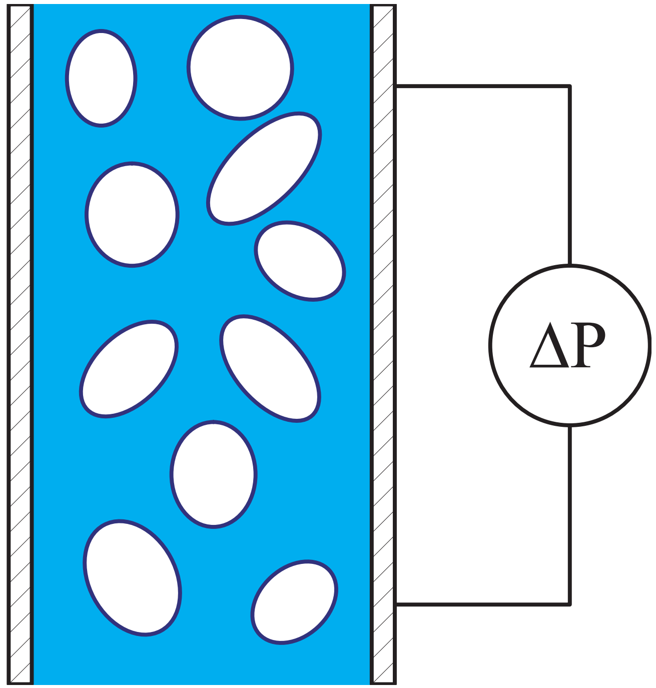
$$\epsilon_{2\phi} \approx \frac{3}{2}\epsilon_D + \left(\epsilon_C - \frac{3}{2}\epsilon_D\right)(1 - R_{D3})^{3/2}$$

VOLUME FRACTION



- Quick closing valves, liquid settling:

$$R_{L3} = \frac{V_L}{V}$$

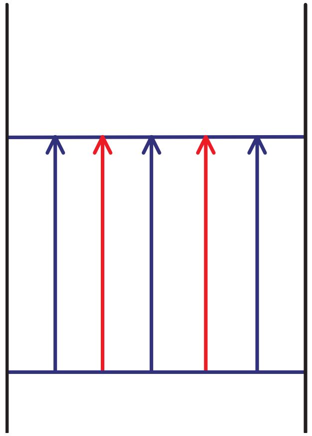


- Gravitational (hydrostatic) pressure drop ($v_L \ll 1$ m/s),

$$\Delta p = \rho g H$$

$$\rho \triangleq \rho_G R_{G3} + \rho_L (1 - R_{G3})$$

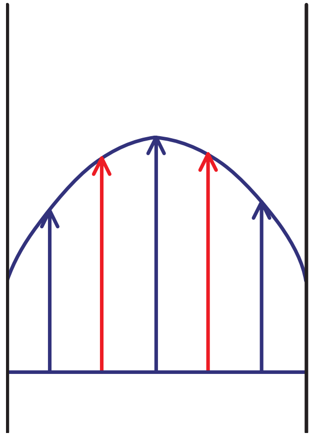
BASIC VOID FRACTION MODELS



1D-1V

Homogeneous

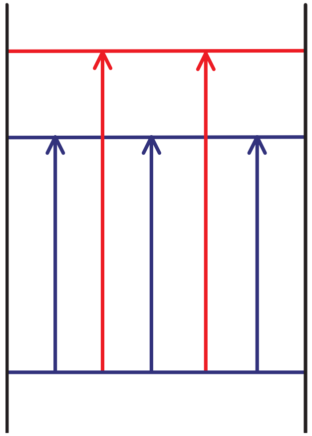
$$\overline{w}_G^X = \overline{w}_L^X = w$$



2D-1V

Bankoff

$$\overline{w}_G^X = \overline{w}_L^X = f(r)$$



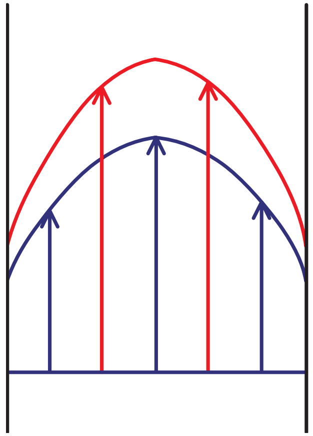
1D-2V

Wallis

$$\overline{w}_G^X = w_G$$

$$\overline{w}_L^X = w_L$$

$$w_L \neq w_G$$



2D-2V

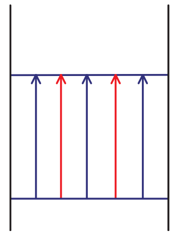
Zuber-Findlay

$$\overline{w}_G^X = f(r)$$

$$\overline{w}_L^X = g(r)$$

- Mechanical equilibrium, force balance, bubbly flow (drag-buoyancy), foam (Marangoni-drag-gravity), not inertia controlled.
- Effect of different gas and liquid velocities: $\overline{w}_G^X \neq \overline{w}_L^X$ (on/off)
- Effet of velocity profiles (on/off).

THE HOMOGENEOUS MODEL (1D-1V)



- **1D-1V**, $\overline{w}_G^X = \overline{w}_L^X = w$
  - What is known: $\overline{Q}_G$, $\overline{Q}_L$.
  - What is unknown: $\overline{R_{G2}} = \nless \alpha_G \ngtr_2$.
- Identical derivation for the 4 models. Mean volume flow rate definition,

$$\overline{Q}_G \triangleq \int_{A_G} w_G \, \mathrm{d}A = \overline{A_G < w_G >_2} = A\overline{R_{G2} < w_G >_2}$$

- Commutativity of averaging operators, uniform velocity profile,

$$\overline{Q}_G = A\overline{R_{G2} < w_G >_2} = A \nless \alpha_G \overline{w}_G^X \ngtr_2 = A\overline{R_{G2}} w_G$$

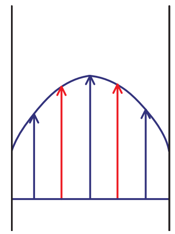
- For the liquid and gas phase,

$$\overline{Q}_L = A(1 - \overline{R_{G2}}) w_L, \quad \overline{Q}_G = A\overline{R_{G2}} w_G$$

- Identical mean velocities,

$$\frac{\overline{Q}_G}{\overline{Q}_L} = \frac{\overline{R_{G2}}}{1 - \overline{R_{G2}}}, \qquad \boxed{\overline{R_{G2}} = \frac{\overline{Q}_G}{\overline{Q}_G + \overline{Q}_L} = \beta}$$

BANKOFF MODEL(2D-1V)



- **2D-1V,** $\overline{w}_G^X = \overline{w}_L^X = w_C \left(\frac{y}{R}\right)^{\frac{1}{m}}, \quad \alpha_G = \alpha_C \left(\frac{y}{R}\right)^{\frac{1}{m}}$
- What is known: $\overline{Q}_G$, $\overline{Q}_L$, what is unknown: $\overline{R_{G2}} = \nless \alpha_G \ngtr_2$.
- Mean volume flow rate definitions,

$$\overline{Q}_G = A \nless \alpha \overline{w}_G^X \ngtr_2 = A f(w_C, \alpha_C, m, n), \; \overline{Q}_L = A \nless (1-\alpha) \overline{w}_L^X \ngtr_2 = A g(w_C, \alpha_C, m, n)$$

- Mean velocity and area fraction,

$$\nless \overline{w}_L^X \ngtr_2 = w_C h(m), \quad \overline{R_{G2}} = \alpha_C k(n)$$

- Eliminate α_C and w_C,

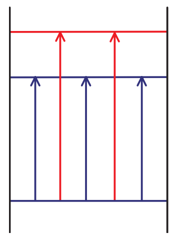
$$\boxed{\overline{R_{G2}} = K\beta}$$

$$K = \frac{2(m+n+mn)(m+n+2mn)}{(n+1)(2n+1)(m+1)(2m+1)}, \quad K = 0,6 \div 1, \; 2 \leqslant m, n \leqslant 7$$

- Closure for steam-water, Bankoff dimensional correlation (p in bar),

$$K = 0,71 + 0,00145p$$

WALLIS MODEL (1D-2V)



- **1D-2V,** $\overline{w}_G^X = w_G$, $\overline{w}_L^X = w_L$, $\alpha_G(r) = \alpha_G$, $w_G \neq w_L$
- What is known: $\overline{Q}_G$, $\overline{Q}_L$, unknown: $\overline{R_{G2}} = \langle\!\langle \alpha_G \rangle\!\rangle_2$
- Mean flow rates definitions,

$$\overline{Q}_G = A \langle\!\langle \alpha \overline{w}_G^X \rangle\!\rangle_2 = A\overline{R_{G2}} w_G$$

$$\overline{Q}_L = A \langle\!\langle (1-\alpha) \overline{w}_L^X \rangle\!\rangle_2 = A(1-\overline{R_{G2}}) w_L$$

- Mean surface fraction,

$$\overline{R_{G2}} = \frac{\overline{Q}_G w_L}{\overline{Q}_L w_G + \overline{Q}_G w_L} = \boxed{\frac{\beta}{1 + \dfrac{(1-\overline{R_{G2}})(w_G - w_L)}{J}}}$$

- Closure for bubbly flows, w_∞, terminal velocity (Clift *et al.* , 1978)

$$w_G - w_L = w_\infty (1 - \overline{R_{G2}}), \quad w_\infty = f(D, \sigma, \rho_L, \rho_G, \mu_L, \cdots)$$

- Wallis diagram (Wallis, 1969), bubble columns, analogy with mass transfer modeling.

SETTING UP THE WALLIS DIAGRAM

- Volumetric flux:

$$j_k \triangleq \alpha_k \overline{w}_k^X = \overline{X_k w_k}, \quad j = j_1 + j_2$$

- Drift velocity: phase relative velocity wrt the center of volume,

$$v_{kj} \triangleq \overline{w}_k^X - j$$

- Drift flux, flux of volume in a frame moving with j,

$$j_{GL} = \alpha_G(\overline{w}_k^X - j)$$

- 1D assumption:

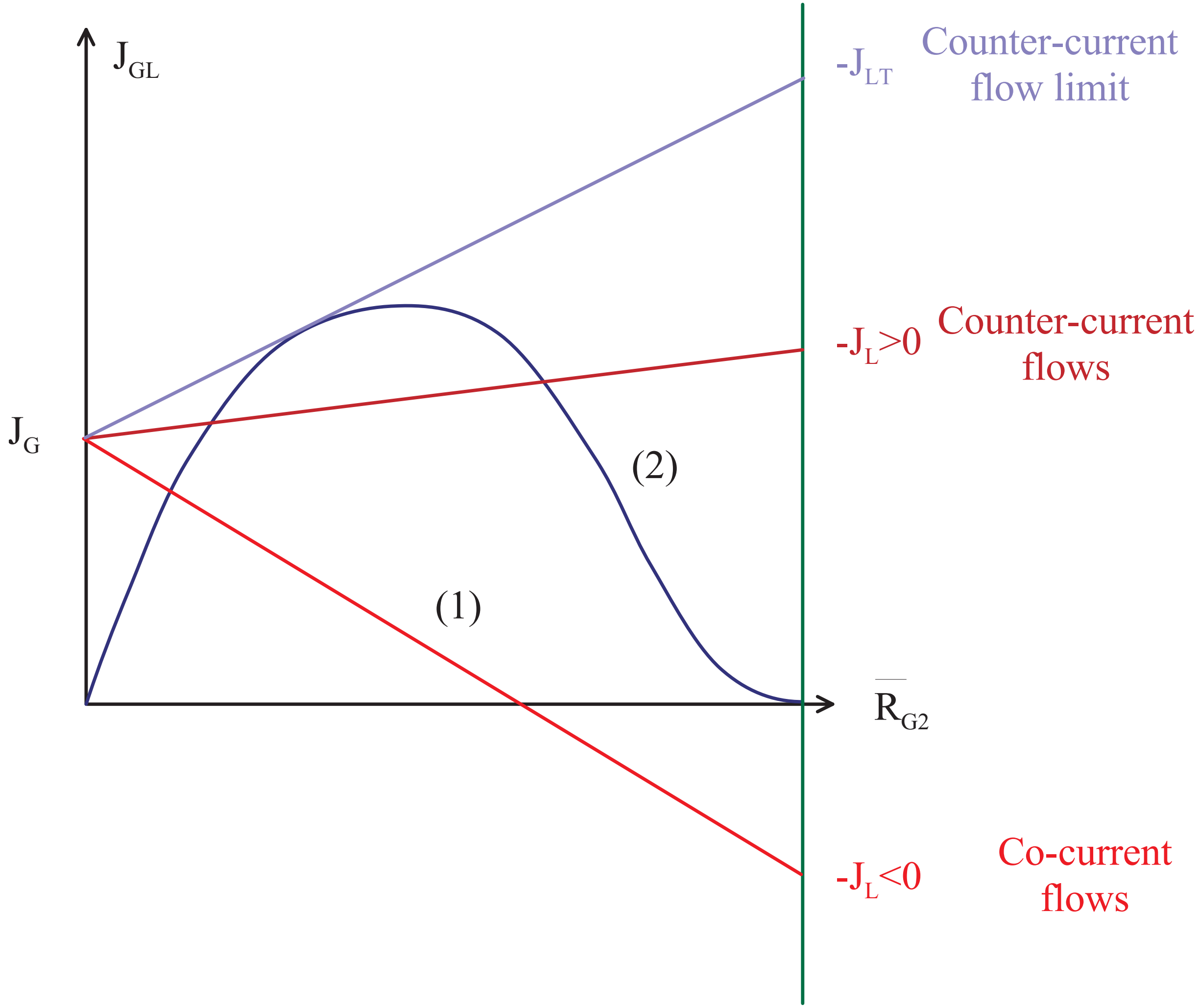
$$J_{GL} = \langle j_{GL} \rangle_2 = \overline{R_{G2}}(w_G - J) = (1 - \overline{R_{G2}})J_G - \overline{R_{G2}}J_L \tag{1}$$

- By definition: $J = J_G + J_L = \overline{R_{G2}} w_G + (1 - \overline{R_{G2}}) w_L$,

$$J_{GL} = \overline{R_{G2}}(1 - \overline{R_{G2}})(w_G - w_L) = w_\infty \overline{R_{G2}}(1 - \overline{R_{G2}})^2 \tag{2}$$

- NB: closure is needed (w_∞), e.g bubbly flows, foam.

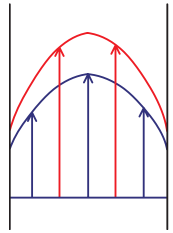
WALLIS DIAGRAM



Bubble columns operation.

- Co-current flow: $J_L > 0$, $J_G > 0$, 1 operating condition.
- Counter-current: $J_L < 0$, $J_G > 0$, 2 operating conditions.
- Counter-current flow limitation, $J_L < -J_{LT}$.

ZUBER & FINDLAY MODEL (2D-2V)



- **2V-2D**: what is known: $\overline{Q}_G$, $\overline{Q}_L$, unknown: $\overline{R_{G2}} = \langle\!\langle \alpha_G \rangle\!\rangle_2$,
- Definition of the local drift velocity,

$$\overline{w}^X_{Gj} = \overline{w}^X_G - j = (1-\alpha_G)(\overline{w}^X_G - \overline{w}^X_L)$$

- Mean drift flux on the cross section,

$$\langle\!\langle \alpha_G \overline{w}^X_{Gj} \rangle\!\rangle_2 = \langle\!\langle \alpha_G \overline{w}^X_G \rangle\!\rangle_2 - \langle\!\langle \alpha_G j \rangle\!\rangle_2$$

- Change of variables for unknown quantities:

$$\widetilde{w}_{GJ} = \frac{\langle\!\langle \alpha_G \overline{w}^X_{Gj} \rangle\!\rangle_2}{\langle\!\langle \alpha_G \rangle\!\rangle_2}, \quad C_0 = \frac{\langle\!\langle \alpha_G j \rangle\!\rangle_2}{\langle\!\langle \alpha_G \rangle\!\rangle_2 \langle\!\langle j \rangle\!\rangle_2}$$

- Previous models are recovered by the Zuber & Findlay model,

$$\boxed{\overline{R_{G2}} = \frac{J_G}{C_0 J + \widetilde{w}_{GJ}}} \quad = \frac{\beta}{C_0 + \frac{\widetilde{w}_{GJ}}{J}}$$

- Zuber & Findlay diagram: $\frac{J_G}{R_G} = C_0 J + \widetilde{w}_{GJ}$

CLOSURES FOR THE ZUBER & FINDLAY MODEL

- ZF diagram, 2 closure relations: C_0, slope, $\widetilde{w}_{GJ}$, y-axis intersection. Flow regime dependent (Ishii, 1977).
- Here $\overline{R_{G2}} \to R_G$ is to be understood,

$$C_0 = \left(1,2 - 0,2\sqrt{\frac{\rho_G}{\rho_L}}\right) \underbrace{(1 - \exp(-18R_G))}_{\text{boiling only}}$$

- Bubbly flows:

$$\widetilde{w}_{GJ} = (C_0 - 1)J + 1,4\left(\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right)^{1/4} (1 - R_G)^{7/4}$$

- Slug flow:

$$\widetilde{w}_{GJ} = (C_0 - 1)J + 0,35\left(\frac{gD(\rho_L - \rho_G)}{\rho_L}\right)^{1/2}$$

ZUBER & FINDLAY MODEL CLOSURES (CT'D)

- Churn flow:

$$\widetilde{w}_{GJ} = (C_0 - 1)J + 1,4\left(\frac{\sigma g(\rho_L - \rho_G)}{\rho_L^2}\right)^{1/4}$$

- Annular flow:

$$\widetilde{w}_{GJ} = \frac{1 - R_G}{R_G + \left(\frac{1+75(1-R_G)}{\sqrt{R_G}}\frac{\rho_G}{\rho_L}\right)^{1/2}}\left(\sqrt{\frac{gD(\rho_L - \rho_G)(1 - R_G)}{0,015\rho_L}}\right)$$

REFERENCE GUIDE, WANT TO KNOW MORE

- Modeling the void fraction: Delhaye (2008).
- Drift flux modeling: Wallis (1969).
- Closures: Ishii (1977), see also the enhanced and revised edition by Ishii & Hibiki (2006).

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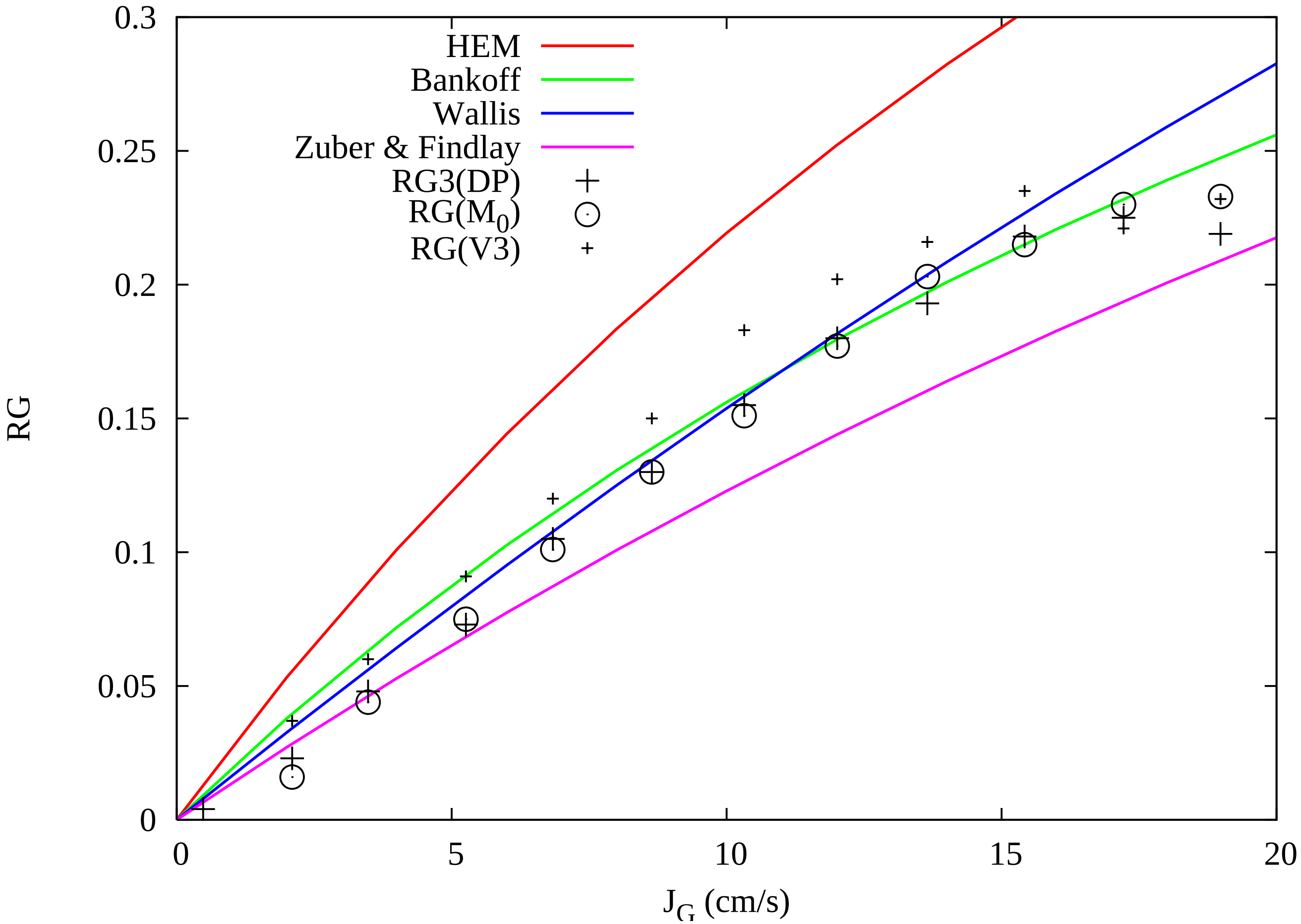
SUGGESTED HOMEWORK ON VOID FRACTION



Objective of the homework: utilize the void fraction models to analyze NMR low liquid velocity data (35 cm/s). Build the Wallis and the Zuber & Findlay diagrams.