A SHORT INTRODUCTION TO TWO-PHASE FLOWS

1D-time averaged models

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TIME- AND AREA-AVERAGED (1D-) MODELS

- Homogeneous model,
- Drift-flux model,
- Two-fluid model,
  - Closure issue,
  - Some unexpected consequences of some modeling assumptions.
    1. Physical consistency of the two-fluid model,
    2. Mathematical nature of the PDE's
- Back to composite averaged equations (common assumptions)

COMPOSITE AVERAGING THE MASS BALANCE

- Space and time averaged mass balance:

$$\frac{\partial}{\partial t} A\overline{R_{k2} < \rho_k >_2} + \frac{\partial}{\partial z} A\overline{R_{k2} < \rho_k w_k >_2} = \Gamma_k, \quad \Gamma_k \triangleq -\overline{\int_{C_i} \dot{m}_k \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}}$$

- Mean value definitions:

$$\overline{R_{k2} < \rho_k >_2} \triangleq \overline{R_{k2}}\rho_k = \alpha_k \rho_k, \quad \overline{R_{k2} < \rho_k w_k >_2} \triangleq \alpha_k \rho_k v_k = \overline{M}_k$$

- With these new variables,

$$\frac{\partial}{\partial t} A\alpha_k \rho_k + \frac{\partial}{\partial z} A\alpha_k \rho_k v_k = \Gamma_k$$

- No assumptions, simple change of variable. $\alpha(\mathbf{r}) \neq \alpha$ and $w_k(\mathbf{r}) \neq v_k$ are non-uniform.

MIXTURE MASS BALANCE

- Add the phase mass balances,

$$\frac{\partial}{\partial t}A(\alpha_1\rho_1 + \alpha_2\rho_2) + \frac{\partial}{\partial z}A(\alpha_1\rho_1 v_1 + \alpha_2\rho_2 v_2) = 0$$

- Mixture density (definition),

$$\rho = \alpha_1\rho_1 + \alpha_2\rho_2$$

- Mixture velocity defined as to preserves the mass flow rate,

$$\rho v = \alpha_1\rho_1 v_1 + \alpha_2\rho_2 v_2$$

- Mixture mass balance,

$$\frac{\partial}{\partial t}A\rho + \frac{\partial}{\partial z}A\rho v = 0$$

MOMENTUM BALANCE

- Simplified, 1 pressure (instead of 3), neglect the effect longitudinal diffusion.

$$\frac{\partial}{\partial t} A\overline{< R_{k2}\rho_k w_k >_2} + \frac{\partial}{\partial z} A\overline{< R_{k2}\rho_k w_k^2 >_2} + A\overline{R_{k2}\frac{\partial}{\partial z} < p_k >_2} - A\overline{R_{k2} < \rho_k g_z >_2}$$

$$= -\overline{\int_{C_i} \left(\dot{m}_k w_k - \mathbf{n}_k \cdot \mathbb{V}_k \cdot \mathbf{n}_z\right) \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} + \overline{\int_{C_k} \mathbf{n}_k \cdot \mathbb{V}_k \cdot \mathbf{n}_z \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}}$$

- 1D assumption: velocity space correlation C, mean pressure, p_k, the so-called *flat profile* assumption.

$$C \triangleq \frac{\overline{< R_{k2}\rho_k w_k^2 >_2}}{\alpha_k \rho_k v_k^2} = 1, \quad \overline{R_{k2}\frac{\partial}{\partial z} < p_k >_2} = \alpha_k \frac{\partial p_k}{\partial z}$$

- Interaction terms, change of variable:

$$-\overline{\int_{C_i} \left(\dot{m}_k w_k\right) \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} = \Gamma_k v_{ki}, \quad \overline{\int_{C_i} \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} = A \nless \gamma \ngtr_2 = A\gamma$$

$$\overline{\int_{C_i} \mathbf{n}_k \cdot \mathbb{V}_k \cdot \mathbf{n}_z \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} = -A\gamma\tau_{ki}, \quad \overline{\int_{C_k} \mathbf{n}_k \cdot \mathbb{V}_k \cdot \mathbf{n}_z \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} = -P_k \tau_{wk}$$

MOMENTUM BALANCE (CT'D)

- New notations, in blue, the flat profile assumption main consequence,

$$\frac{\partial}{\partial t} A\alpha_k\rho_k v_k + \frac{\partial}{\partial z} A\alpha_k\rho_k v_k^2 - A\alpha_k \frac{\partial p_k}{\partial z} = \Gamma_k v_{ki} - A\gamma\tau_{ki} - P_k\tau_{wk} \cdots$$

- Momentum balance for the mixture, single pressure

$$\frac{\partial}{\partial t} A(\alpha_1\rho_1 v_1 + \alpha_2\rho_2 v_2) + \frac{\partial}{\partial z} A(\alpha_1\rho_1 v_1^2 + \alpha_2\rho_2 v_2^2) - A\frac{\partial p}{\partial z} = -P\tau_w$$

- Another form of the inertia term, $x \triangleq \frac{M_G}{M}$,

$$\alpha_G\rho_G v_G^2 + \alpha_L\rho_L v_L^2 = \frac{G^2}{\rho'}, \quad \frac{1}{\rho'} = \frac{x^2}{\alpha\rho_G} + \frac{(1-x)^2}{(1-\alpha)\rho_L}, \quad G = \frac{M}{A}.$$

- Give two examples of inconsistency of the flat profile assumption.

TOTAL ENERGY BALANCE

- Energy balance, native form $\longrightarrow$ enthalpy form,

$$\frac{\partial}{\partial t} A_k < \rho_k \left(u_k + \frac{1}{2} \mathrm{v}_k^2 \right) >_2 + \frac{\partial}{\partial z} A_k < \rho_k w_k \left(u_k + \frac{1}{2} \mathrm{v}_k^2 \right) >_2$$

$$+ \frac{\partial}{\partial z} A_k < \mathbf{n}_z \cdot (\mathbf{q}_k - \mathbb{T}_k \cdot \mathbf{v}_k) >_2 - A_k < \rho_k \mathbf{g}_k \cdot \mathbf{v}_k >_2$$

$$= - \int_{C_i \cup C_k} \left(\dot{m}_k \left(u_k + \frac{1}{2} \mathrm{v}_k^2 \right) + \mathbf{n}_k \cdot (\mathbf{q}_k - \mathbb{T}_k \cdot \mathbf{v}_k) \right) \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}$$

- In the $\frac{\partial}{\partial z}$ terms, $u_k \rightarrow h_k - p_k/\rho_k$, $\mathbb{T}_k \rightarrow \mathbb{V}_k$,
- In the $\frac{\partial}{\partial t}$ term $u_k \rightarrow h_k - p_k/\rho_k$ adds $-\frac{\partial}{\partial t} A_k < p_k >_2$, use the identity (2),

$$\frac{\partial}{\partial t} A_k < p_k >_2 = A_k < \frac{\partial p_k}{\partial t} >_2 + \int_{C_i \cup C_k} p_k \mathbf{v}_i \cdot \mathbf{n} \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}$$

- Collect the pressure terms in the RHS,

TOTAL ENERGY BALANCE (CT'D)

- Energy balance in enthalpy form,

$$\frac{\partial}{\partial t}A_k < \rho_k \left(h_k + \frac{1}{2}\mathrm{v}_k^2\right) >_2 - A_k < \frac{\partial p_k}{\partial t} >_2 + \frac{\partial}{\partial z}A_k < \rho_k w_k \left(h_k + \frac{1}{2}\mathrm{v}_k^2\right) >_2$$

$$+\frac{\partial}{\partial z}A_k < \mathbf{n}_z \cdot (\mathbf{q}_k - \mathbb{V}_k \cdot \mathbf{v}_k) >_2 - A_k < \rho_k \mathbf{g}_k \cdot \mathbf{v}_k >_2$$

$$= -\int_{C_i \cup C_k} \left(\dot{m}_k \left(h_k + \frac{1}{2}\mathrm{v}_k^2\right) + \mathbf{n}_k \cdot (\mathbf{q}_k - \mathbb{V}_k \cdot \mathbf{v}_k)\right) \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}$$

- Neglect the diffusive term, $\mathrm{v}_k^2 \approx v_k^2$, the mean enthalpy preserves the flux,

$$\frac{\partial}{\partial t}A\alpha_k\rho_k \left(h_k + \frac{1}{2}v_k^2\right) - A\alpha_k \frac{\partial p_k}{\partial t} + \frac{\partial}{\partial z}A\alpha_k\rho_k v_k \left(h_k + \frac{1}{2}v_k^2\right)$$

$$-A\alpha_k\rho_k g_k v_k = \Gamma_k h_{ki}^t + A\gamma q_{ki} + P_k q_{kw}$$

MIXTURE ENERGY BALANCE

- Add the two phase balances, interaction terms sum vanish at the interface,

$$\frac{\partial}{\partial t} A(\alpha_1 \rho_1 h_1^t + \alpha_2 \rho_2 h_2^t) + \frac{\partial}{\partial z} A(\alpha_1 \rho_1 v_1 h_1^t + \alpha_2 \rho_2 v_2 h_2^t)$$

$$-A \frac{\partial p}{\partial t} - A(\alpha_1 \rho_1 g_1 v_1 + \alpha_2 \rho_2 g_2 v_2) = P q_w,$$

- where the total enthalpy is $h_k^t \triangleq h_k + \frac{1}{2} v_k^2$,
- Other practical form of the enthalpy flux,

$$\underbrace{A\alpha_V \rho_V v_V}_{M_V} h_V^t + \underbrace{A\alpha_L \rho_L v_L}_{M_L} h_L^t = M(x h_V^t + (1-x) h_L^t)$$

THE HOMOGENEOUS MODEL AT THERMODYNAMIC EQUILIBRIUM (HEM)

- 3 balances for the mixture + 3 assumptions
  - Mean velocity are equal : $w_V = w_L \equiv \alpha = \beta$
  - Mean temperatures satisfy the equilibrium condition : $T_L = T_V = T_{\mathrm{sat}}(p)$
- Thermodynamique, EOS,

$$\rho_L = \rho_{L\mathrm{sat}}(p), \quad \rho_V = \rho_{V\mathrm{sat}}(p), \qquad h_L = h_{L\mathrm{sat}}(p), \quad h_V = h_{V\mathrm{sat}}(p)$$

- HEM void fraction,

$$\alpha = \beta = \frac{Q_G}{Q_G + Q_L} = \frac{x\rho_L}{x\rho_L + (1-x)\rho_V} = \alpha(x,p)$$

- Balance equations are identical to that of single-phase flow,

$$\frac{\partial}{\partial t}A\rho + \frac{\partial}{\partial z}A\rho w = 0, \quad \rho = \alpha\rho_V + (1-\alpha)\rho_L$$

$$\frac{\partial}{\partial t}A\rho w + \frac{\partial}{\partial z}A\rho w^2 + A\frac{\partial p}{\partial z} = -P\tau_W + A\rho g_z$$

$$\frac{\partial}{\partial t}A\rho(h + \frac{1}{2}w^2) - A\frac{\partial p}{\partial t} + \frac{\partial}{\partial z}A\rho w(h + \frac{1}{2}w^2) = Pq_W + A\rho g_z w$$

HEM (CT'D)

- Other practical form, combine with the mass balance,

$$\frac{\partial}{\partial t} A\rho + \frac{\partial}{\partial z} A\rho w = 0$$

$$\rho \frac{\partial w}{\partial t} + \rho w \frac{\partial w}{\partial z} + \frac{\partial p}{\partial z} = -\frac{P}{A}\tau_W + \rho g_z$$

$$\rho \frac{\partial}{\partial t}(h + \frac{1}{2}w^2) - \frac{\partial p}{\partial t} + \rho w \frac{\partial}{\partial z}(h + \frac{1}{2}w^2) = \frac{P}{A} q_W + \rho g_z w$$

- Mechanical energy balance, momentum balance $\times w$,

$$\rho \frac{\partial}{\partial t}\frac{1}{2}w^2 + \rho w \frac{\partial}{\partial z}\frac{1}{2}w^2 + w\frac{\partial p}{\partial z} = -\frac{P}{A} w\tau_W + \rho g_z w$$

- Entropy balance, $T\mathrm{d}s = \mathrm{d}h - \dfrac{\mathrm{d}p}{\rho}$

$$\rho T \frac{\partial s}{\partial t} + \rho w T \frac{\partial s}{\partial z} = \frac{P}{A}(q_W + w\tau_W)$$

HEM (CT'D)

- Alternate form with total enthalpy and entropy,

$$\frac{\partial}{\partial t}\rho + w\frac{\partial \rho}{\partial z} + \rho\frac{\partial w}{\partial z} = -\frac{\rho w}{A}\frac{\mathrm{d}A}{\mathrm{d}z}$$
$$\rho\frac{\partial}{\partial t}(h + \frac{1}{2}w^2) - \frac{\partial p}{\partial t} + \rho w\frac{\partial}{\partial z}(h + \frac{1}{2}w^2) = \frac{P}{A}q_W + \rho g_z w$$
$$\rho T\frac{\partial s}{\partial t} + \rho w T\frac{\partial s}{\partial z} = \frac{P}{A}(q_W + w\tau_W)$$

- Very important particular case: stationary flow, adiabatic, no friction nor volume forces,

$$M = A\rho w = \mathrm{cst}$$
$$h + \frac{1}{2}w^2 = \mathrm{cst}$$
$$s = x s_V + (1-x)s_L = \mathrm{cst}$$

- Applications: flashing in long pipes, w/o heating, critical flow (no model !).

CLOSURES FOR THE HEM

- Friction and heat flux, q_W, τ_W,
- Independent variables : x, w, et p,
- EOS, $v = \dfrac{1}{\rho} = xv_V + (1-x)v_L$, specific volume [m^3/kg],

$$v_L = v_{L\mathrm{sat}}(p), \quad v_V = v_{V\mathrm{sat}}(p), \qquad h_L = h_{L\mathrm{sat}}(p), \quad h_V = h_{V\mathrm{sat}}(p)$$

$$v_{V,p}, \quad v_{L,p}, \quad h_{V,p}, \quad h_{L,p}$$

- NB: back again to the thermodynamic consistency issue.

DRIFT-FLUX MODEL

- Different phase mean velocities, "mechanical non-equilibrium".
- Mass balances for the liquid and vapor, momentum balance for the mixture, $w_V \neq w_L$, w_m mixture velocity.
- Additional closure (core modeling, FLICA)

$$w_V - w_L = f(x, p, \alpha, \text{flow regime}, \cdots)$$

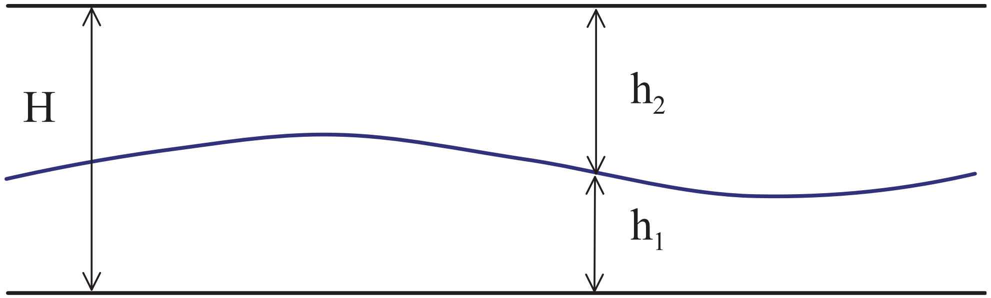
$$J_{GL} = f(\alpha, \text{flow regime}, \cdots)$$

- Slow transients NOT inertia controlled.
- Can also be used in 3D, see for example Delhaye (2008a), Ishii & Hibiki (2006).
- Main advantage: only one momentum balance.

THE TWO-FLUID MODEL

- Mechanical and thermal non-equilibriums,
  - 3 balance equations per phase (6), or
  - 3 balance equations for the mixture and 3 equations for the dispersed phase.
- Closures
  - Topological relations, $< pq >$, $< p >< q >$, the pressure issue,
  - Interactions at the interface,
  - Interactions of each phase at the wall.
- Consequences of the closure assumptions,
  - Propagation characteristics,
  - Critical flow,
  - Mathematical nature of the PDE's (hyperbolicity ?).

EXAMPLE: STRATIFIED FLOWS



- Isothermal, incompressible, horizontal, $\mu = 0$, $\sigma = 0$, $\dot{m} = 0$, 2D.
- Mass balance,

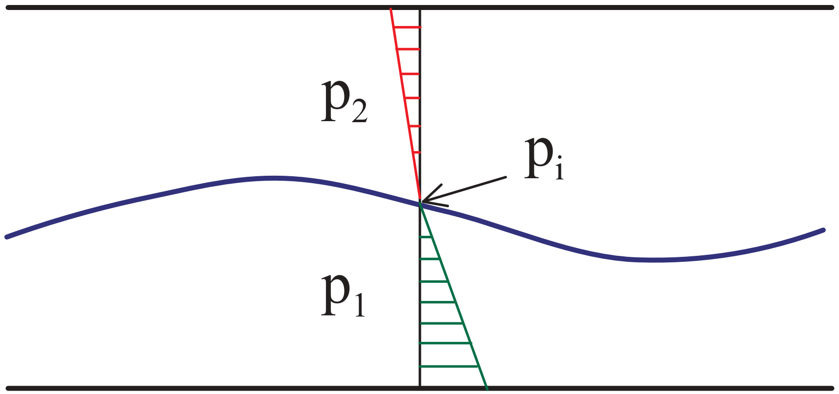
$$\frac{\partial}{\partial t} h_1 \rho_1 + \frac{\partial}{\partial z} h_1 \rho_1 < u_1 >= 0$$
$$\frac{\partial}{\partial t} h_2 \rho_2 + \frac{\partial}{\partial z} h_2 \rho_2 < u_2 >= 0$$

- Momentum balances, jump of momentum at the interface,

$$\frac{\partial}{\partial t} h_1 \rho_1 < u_1 > + \frac{\partial}{\partial z} h_1 \rho_1 < u_1^2 > + \frac{\partial}{\partial z} h_1 < p_1 >= p_{i1} \frac{\partial h_1}{\partial z}$$
$$\frac{\partial}{\partial t} h_2 \rho_2 < u_2 > + \frac{\partial}{\partial z} h_2 \rho_2 < u_2^2 > + \frac{\partial}{\partial z} h_2 < p_2 >= p_{i2} \frac{\partial h_2}{\partial z}$$
$$p_{i1} = p_{i2} \triangleq p_i$$

CLOSURES

- 4 equations, 5 unknown variables h, $< u_1 >$, $< u_2 >$, $< p_1 >$, $< p_2 >$,
- 3 unknown quantities: $< u_1^2 >$, $< u_2^2 >$, p_i.



- Topological relation,

$$< p_1 >= p_i + \frac{1}{2}\rho_1 g h_1$$
$$< p_2 >= p_i - \frac{1}{2}\rho_2 g h_2$$

- Cannot be derived from momentum $\perp$.

- Spatial correlations, flat profile assumption, or relaxation

$$\frac{< u_k^2 >}{< u_k >^2} = 1, \quad \frac{\mathrm{d}}{\mathrm{d}t} < u_k^2 >= \frac{1}{T}\left[< u_k^2 > - < u_k^2 >_0\right]$$

- Closed system.

STABILITY OF STRATIFIED FLOW

- Solve PDE's, $\mathbf{A}\frac{\partial \mathbf{X}}{\partial t} + \mathbf{B}\frac{\partial \mathbf{X}}{\partial z} = 0$, $\mathbf{X} = (h, u_1, u_2, p_1)$:

$$\mathbf{A} \triangleq \begin{bmatrix} \rho_1 & 0 & 0 & 0 \\ -\rho_2 & 0 & 0 & 0 \\ 0 & \rho_1 h_1 & 0 & 0 \\ 0 & 0 & \rho_2 h_2 & 0 \end{bmatrix}, \quad \mathbf{B} \triangleq \begin{bmatrix} \rho_1 u_1 & \rho_1 h_1 & 0 & 0 \\ -\rho_2 u_2 & 0 & \rho_2 h_2 & 0 \\ \frac{1}{2}\rho_1 g h_1 & \rho_1 u_1 h_1 & h_1 & \\ (\rho_1 - \frac{1}{2}\rho_2) g h_2 & 0 & \rho_2 u_2 h_2 & h_2 \end{bmatrix}$$

- Use the perturbation method, Van Dyke (1975) : $\mathbf{X} = \mathbf{X}_0 + \epsilon \mathbf{X}_1 + \mathcal{O}(\epsilon^2)$,
- Linearizes the PDE's, separate the orders,

$$\mathbf{A}\frac{\partial \mathbf{X}_0}{\partial t} + \mathbf{B}\frac{\partial \mathbf{X}_0}{\partial z} = 0, \quad \mathbf{X}_0 = \text{cst}$$

$$\mathbf{A}(\mathbf{X}_0)\frac{\partial \mathbf{X}_1}{\partial t} + \mathbf{B}(\mathbf{X}_0)\frac{\partial \mathbf{X}_1}{\partial z} = 0$$

- $\mathbf{X}_0$, base solution, $\mathbf{X}_1$, first order (linear) perturbation,
- $z \in [a, b]$, BC and IC are needed.

STABILITY OF STRATIFIED FLOW (CT'D)

- Progressive waves, $\mathbf{X}_1 = \widetilde{\mathbf{X}}_1 \exp i(\omega t - kz)$, $c = \omega/k$, phase velocity,
- Temporal stability, $X_1(a,t) = f(t)$, $\omega \in \mathbb{R}$, how (far) does perturbations propagates into the domain ?
- Spatial stability: $X_1(z,0) = g(z)$, $k \in \mathbb{R}$, perturbation amplification?
- When the RHS of balance equations are non-zero, long wave assumption.

$$(c\mathbf{A}(\mathbf{X}_0) - \mathbf{B}(\mathbf{X}_0))\widetilde{\mathbf{X}}_1 = 0$$

- One class of solution, $\widetilde{\mathbf{X}}_1 \in \ker(c\mathbf{A}(\mathbf{X}_0) - \mathbf{B}(\mathbf{X}_0))$
- Dispersion equation,

$$-\rho_1\rho_2 h_1 h_2 \left(\rho_1 h_2 (u_1 - c)^2 + \rho_2 h_1 (u_2 - c)^2 - (\rho_1 - \rho_2) g h_1 h_2\right) = 0$$

- Stable if and only if the 2 roots are real,

$$(u_1 - u_2)^2 \leqslant g(\rho_1 - \rho_2)\frac{\rho_1 h_2 + \rho_2 h_1}{\rho_1 \rho_2}$$

STABILITY OF STRATIFIED FLOW (CT'D)

- Conditional stability : $\Delta u \leqslant \Delta u_C$

$$(u_1 - u_2)^2 \leqslant g(\rho_1 - \rho_2)\frac{\rho_1 h_2 + \rho_2 h_1}{\rho_1 \rho_2}$$

  – Kelvin-Helmholtz instability,

  – Heavy on top, light below, $\rho_2 > \rho_1$, always unstable (hopefully).

  – Flat pressure profiles, ($g = 0$), always unstable.

- Nature of PDE's: conditionally hyperbolic,
- $g = 0$, the IC problem is ill-posed (Hadamard) $\Rightarrow$ No possibility to get a stationary state from a transient calculation.
- With no differential terms in the closures, the two-fluid model with one pressure leads to ill-posed problems.
- Why codes produce a solution ?

PRESSURE DROP MODELING

- Simplified flow model,
  - Mass balance of the mixture,
  - Momentum balance of the mixture,
  - If adiabatic: $x = x_0$, or solve the energy equation,
  - Evolution equation $(w_V \neq w_L)$,
- Closures :

$$\overline{\sum_{L,G} \int_{C_k} \mathbf{n}_k \cdot \mathbb{V}_k \cdot \mathbf{n}_z \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} = -P\tau_W$$

$$= -\sum_{L,G} \overline{\int_{C_k} \mathbf{n}_k \cdot \mathbf{q}_k \frac{\mathrm{d}l}{\mathbf{n}_k \cdot \mathbf{n}_{kC}}} = Pq_W$$

- NB: Constant pipe cross-sectional area, $A = \text{cst}$, $\mathbf{n}_k \cdot \mathbf{n}_{kC} = 1$

PRESSURE DROP MODELING (CT'D)

- Stationary flow, constant flow area,

$$\frac{\mathrm{d}}{\mathrm{d}z}\rho w = 0, \quad G = \mathrm{cst}$$

$$\frac{\mathrm{d}}{\mathrm{d}z}\rho w^2 + \frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{P}{A}\tau_W + \rho g_z, \quad \frac{P}{A} \triangleq \frac{4}{D_h}$$

- Wall friction appears only in the momentum balance,

$$\frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{\mathrm{d}}{\mathrm{d}z}\rho w^2 - \frac{P}{A}\tau_W + \rho g_z \triangleq \left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_A + \left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_F + \left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_G$$

- Experiments where $\frac{\mathrm{d}p}{\mathrm{d}z}$ and possibly $\alpha = \overline{R_{G2}}$ are measured.
- NB: evolution equation is used for $\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_A$:

Use the models with the same set of assumptions.

WALL FRICTION WITH THE HEM

- Friction should not be dominant,

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_F = -\frac{P}{A}\tau_W = -\frac{4}{D_h}\tau_W$$

- NB: frictional pressure drop, $f = 4C_f$ (quite tricky...)

$$C_F = \frac{\tau_W}{\frac{1}{2}\rho w^2}, \quad f = \frac{D\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_F}{\frac{1}{2}\rho w^2}$$

1. Annular flow $x \approx 1$, $C_F = 0,005$,
   flashing flows, $x \approx 0$, $C_F = 0,003$.
2. $x \ll 1$, $C_F = C_{FL}$, $M = M_L + M_V$,
   $x \approx 1$, $C_F = C_{FG}$, $M = M_L + M_V$
3. NB: Single-phase friction factors,

$$\text{Poiseuille} : \frac{16}{\text{Re}}, \quad \text{Blasius} : \begin{cases} 0,079\ \text{Re}^{-0,25}, & \text{Re} < 20\ 000 \\ 0,046\ \text{Re}^{-0,20}, & \text{Re} > 20\ 000 \end{cases} \qquad \text{Re} \triangleq \frac{GD}{\mu}$$

WALL FRICTION (CT'D)

- Historical perspective, "two-phase viscosity",
  - Dukler (1964) : $\mu = \beta\mu_g + (1-\beta)\mu_L$, $C_F = 0,0014 + 0,125\mathrm{Re}^{-0,32}$
  - Ishii-Zuber (1978), (liquid-liquid or gas-liquid, $\alpha_{DM} = 0,62$)

$$\frac{\mu}{\mu_C} = \left(1 - \frac{\alpha_D}{\alpha_{DM}}\right)^{-2,5\alpha_{DM}\frac{\mu_D + 0,4\mu_C}{\mu_D + \mu_C}}$$

- Acceleration pressure drop, use the appropriate evolution ($\alpha \neq \beta$),

$$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_A = -G^2 \frac{\mathrm{d}}{\mathrm{d}z}\left[\frac{x^2}{\alpha\rho_V} + \frac{(1-x)^2}{(1-\alpha)\rho_L}\right]$$

- Quality from the enthalpy balance, low velocity, thermal equilibrium,

$$G\frac{\mathrm{d}}{\mathrm{d}z}(xh_V + (1-x)h_L) = \frac{4}{D_c}q_W$$

TWO-COMPONENT, LOCKHART & MARTINELLI

- Air-water experiments, low pressure, Δp_F et R_{G3} are measured (QCV). Three sets of experiments in the same horizontal pipe.

Conditions	two-phase	gas only	liquid only
Mass rate	$M = M_G + M_L$	M_G	M_L
$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_F$	$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)$	$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_G$	$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_L$

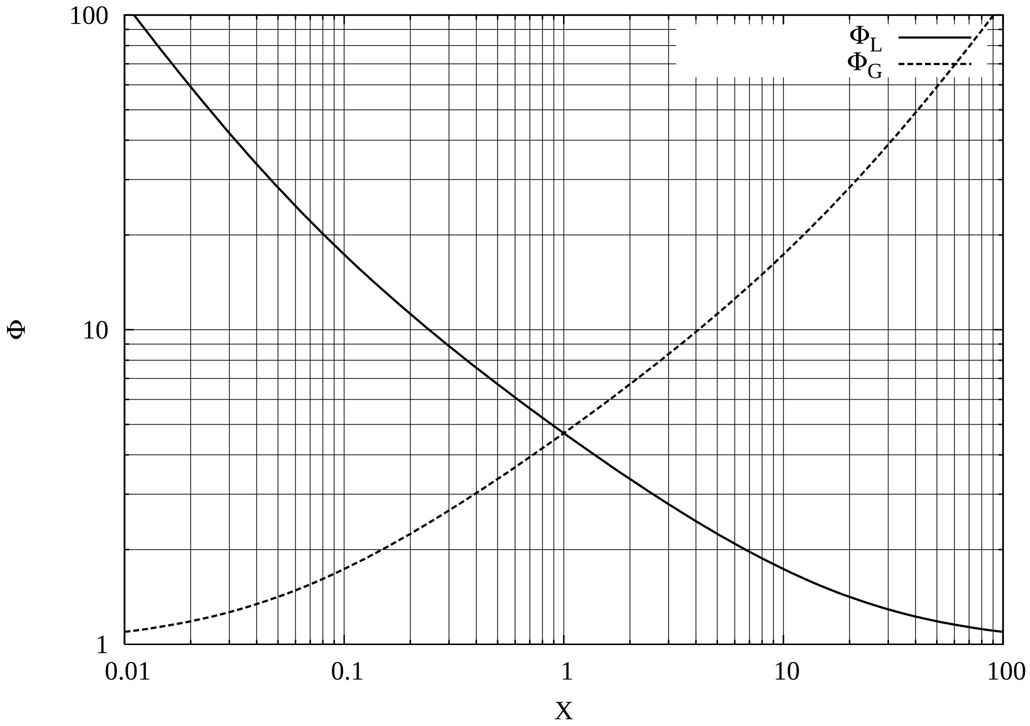
- Definitions on the non-dimensional friction pressure drop: (*two-phase pressure drop multiplier*)

$$\Phi_L^2 \triangleq \frac{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)}{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_L}, \quad \Phi_G^2 \triangleq \frac{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)}{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_G}, \quad X^2 \triangleq \frac{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_L}{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_G}$$

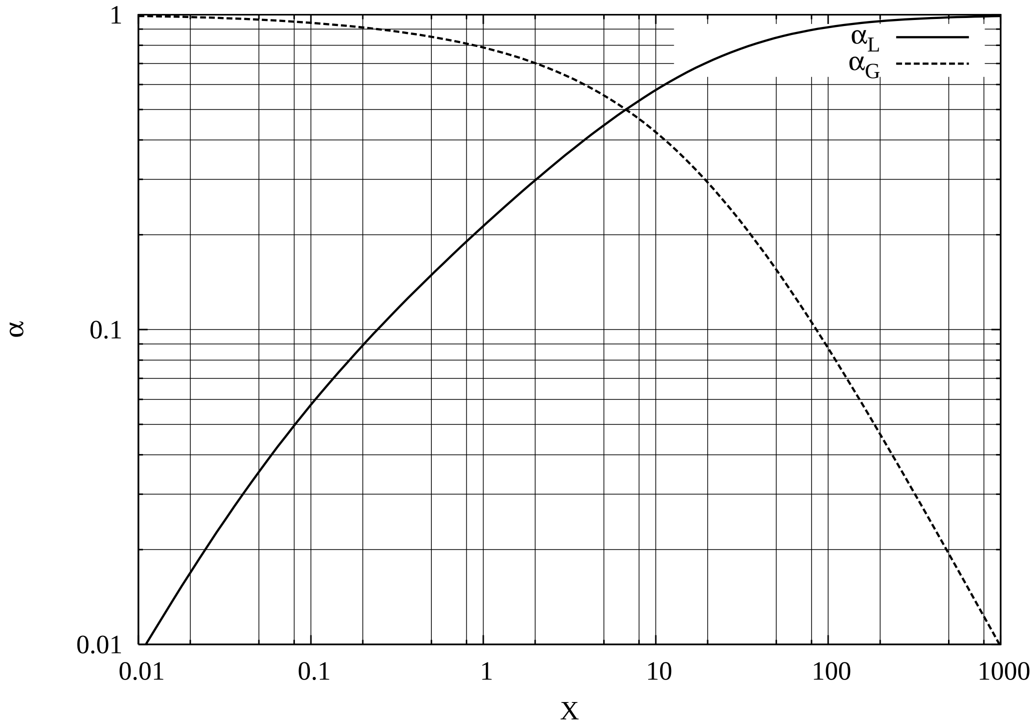
- Blasius is used ($C_f = 0,046 \ \mathrm{Re}^{-0,2}$), X, L. & M. parameter

$$X_{tt} = \left(\frac{\mu_L}{\mu_G}\right)^{0,1} \left(\frac{1-x}{x}\right)^{0,9} \left(\frac{\rho_G}{\rho_L}\right)^{0,5}$$

LOCKHART & MARTINELLI CORRELATION



$$\Phi_L = \sqrt{1 + \frac{20}{X} + \frac{1}{X^2}}$$



$$\alpha_L = \frac{X}{\sqrt{1 + 20X + X^2}}$$

STEAM-WATER, MARTINELLI & NELSON

- Steam water experiments, $34.5 \div 207$ bar, $\Delta p_G \approx 0$.
  Two experiments in the same helical tube. $\Delta p = \Delta p_F + \Delta p_A$.

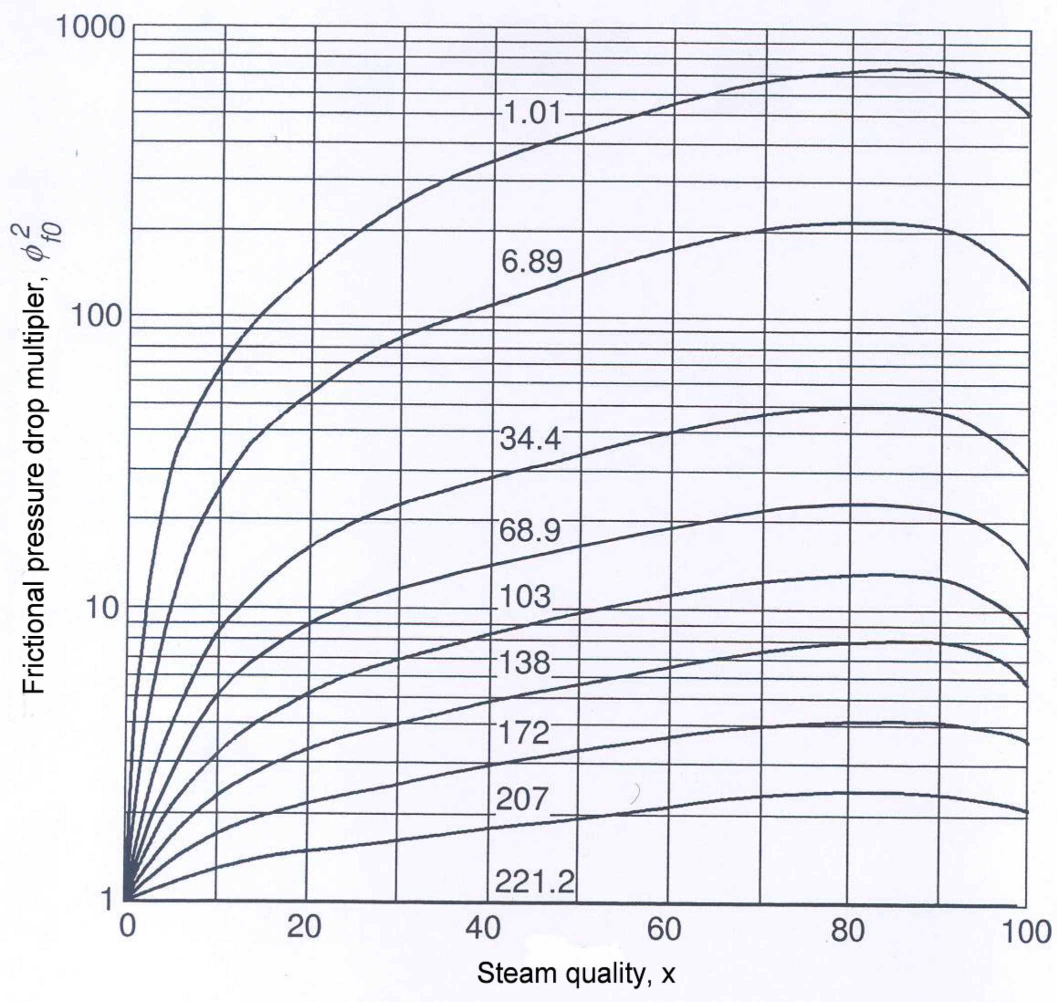
Conditions	two-phase	liquid only
Mass rate	M	$M_L = M$
$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_F$	$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)$	$\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{fo}$

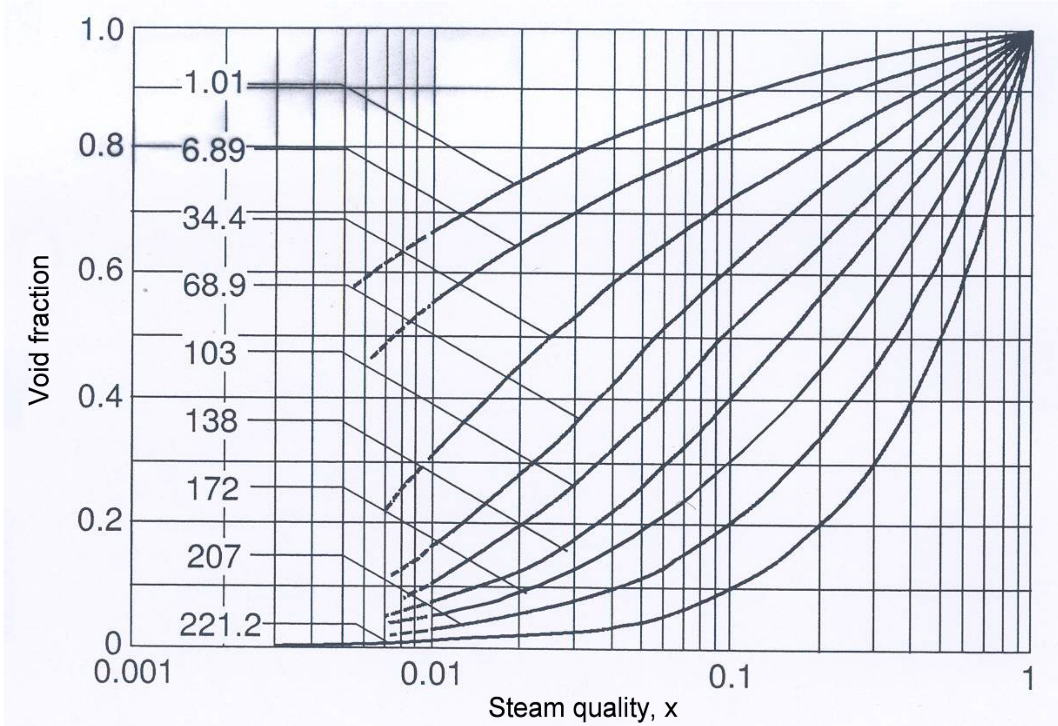
- Two-phase multiplier,

$$\Phi_{f0}^2 = \frac{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)}{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{fo}}$$

- Evolution equation (acceleration pressure drop), data and models.

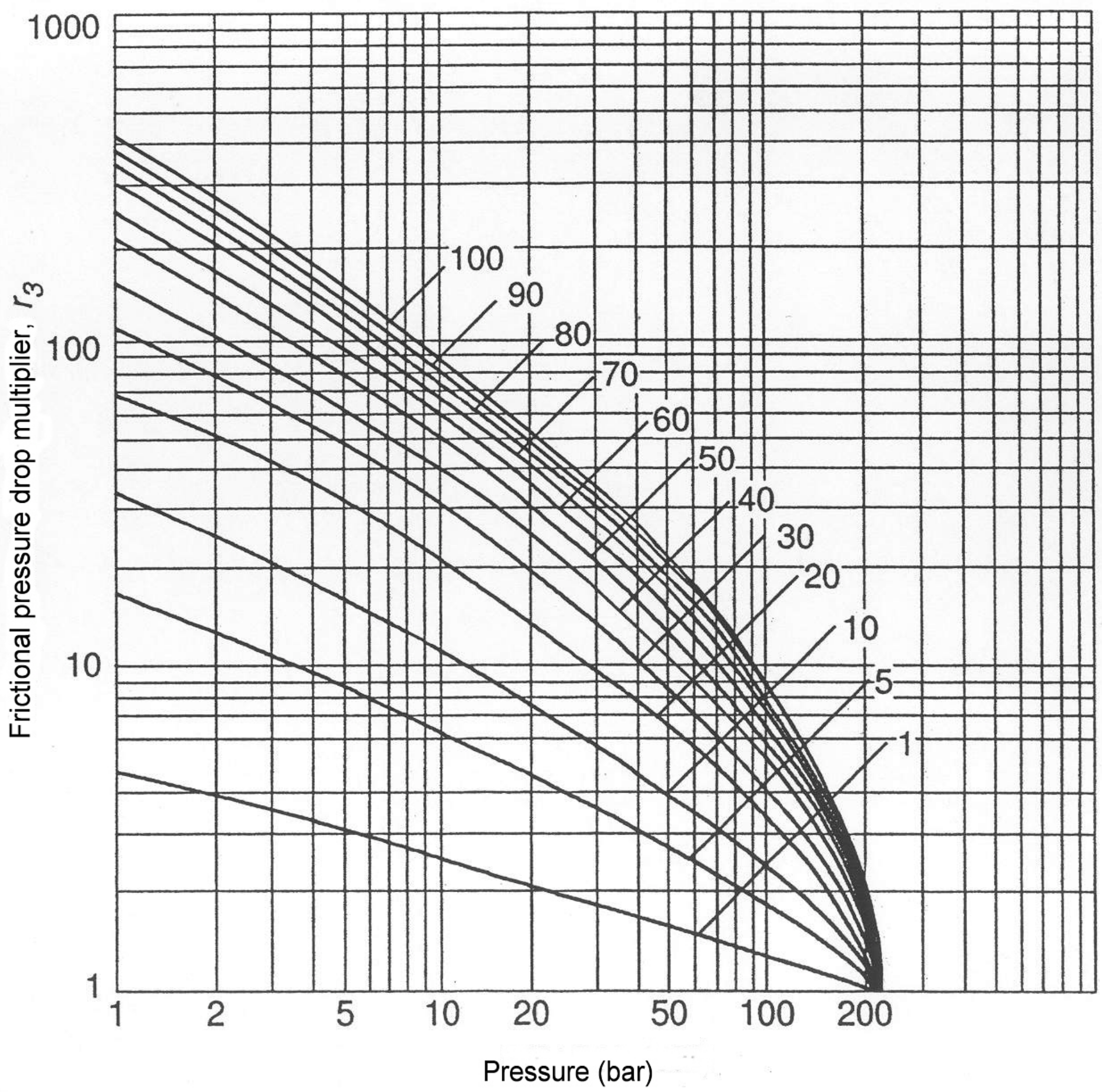
MARTINELLI & NELSON CORRELATIONS





Void fraction vs quality and pressure (bar)

$$\Phi^2_{f0} = \frac{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)}{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{fo}}$$



Friction multiplier vs pressure and quality (%).

- Boiling flows, Thom.

$$r_3 = \frac{\Delta p_F}{\Delta p_{Fo}} = \frac{1}{x_S} \int_0^{x_S} \Phi_{Lo}^2 \mathrm{d}x$$

- Other methods, see Delhaye (2008b).

FRIEDEL'S CORRELATION

- Thousands of data reduction, various fluids, non-dimensional. Arbitrary orientation. Two-phase multiplier by Martinelli & Nelson. M and x are given,

$$\Phi^2_{Lo} = \frac{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_F}{\left(\frac{\mathrm{d}p}{\mathrm{d}z}\right)_{Lo}} = E + \frac{3,24FH}{\mathrm{Fr}^{0,045}\mathrm{We}^{0,035}}$$

$$\rho_h = \left(\frac{x}{\rho_G} + \frac{1-x}{\rho_L}\right)^{-1}, \quad \mathrm{We} = \frac{G^2 D}{\sigma \rho_h}, \quad \mathrm{Fr} = \frac{G^2}{gD\rho_h^2}$$

$$H = \left(\frac{\rho_L}{\rho_G}\right)^{0{,}91}\left(\frac{\mu_G}{\mu_L}\right)^{0{,}19}\left(1 - \frac{\mu_G}{\mu_L}\right)^{0{,}7}, \quad F = x^{0,78}(1-x)^{0,224}$$

$$C_{FGo} = C_{FG}(M), \quad C_{FLo} = C_{FL}(M), \quad E = (1-x)^2 + x^2\frac{\rho_L C_{FGo}}{\rho_G C_{FLo}}$$

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