A SHORT INTRODUCTION TO TWO-PHASE FLOWS

Critical flow phenomenon

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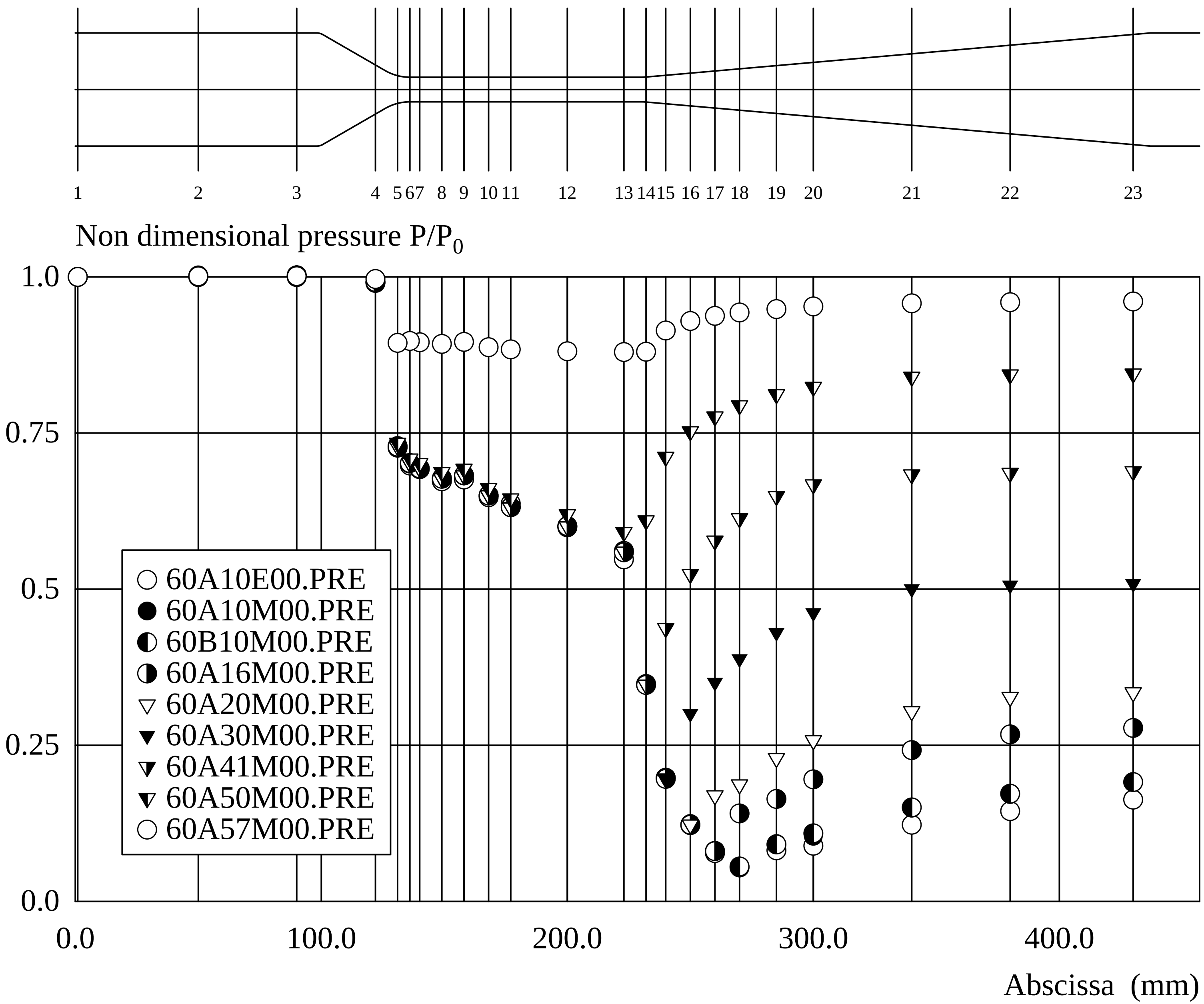
INDUSTRIAL OCCURRENCE

- Depressurization of a nuclear reactor, LOCA (small or large break)
- Industrial accidents prevention
  - Safety valves sizing, SG, chemical reactor.
  - liquid helium storage in case of vacuum loss.
  - LPG storage in case of fire.
- Two typical situations,
  - A pressurized liquid becomes super-heated due to the break, flashing occurs.
  - A gas is created in a vessel, exothermal chemical reaction, pressurizes the vessel, thermal quenching to recover control.
- Critical flow: for given reservoir conditions (pressure), and varying outlet conditions, there exists a limit to the flow rate that can leave the system.

SUMMARY

- Two-component flows
  - Experimental characterization
  - Geometry and inlet effects
- Steam water flows, saturation and subcooling
- Theory and modeling, 2 particular simple cases,
  - Single-phase flow of a perfect gas
  - Two-phase flow at thermodynamic equilibrium
  - General theory, if time permits...

SINGLE-PHASE GAS FLOW, LONG NOZZLE



File	M_G kg/h	P_{back} bar
60A10E00.PRE	363.9	0.973
60A10M00.PRE	364.3	1.127
60B10M00.PRE	362.9	1.135
60A16M00.PRE	364.6	1.650
60A20M00.PRE	364.5	1.986
60A30M00.PRE	364.1	3.023
60A41M00.PRE	364.4	4.088
60A50M00.PRE	361.3	5.022
60A57M00.PRE	246.6	5.749

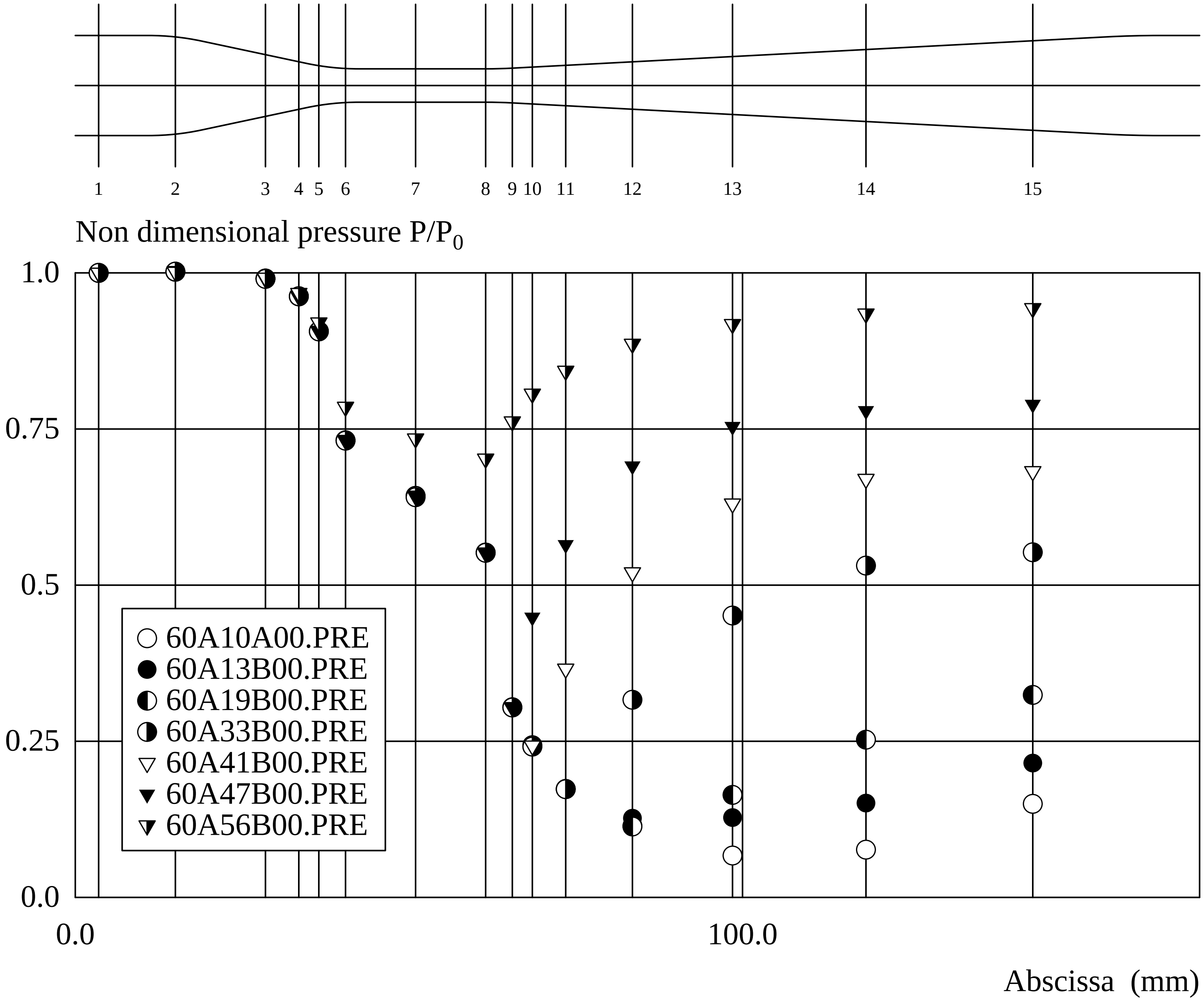
air, $T_G \approx 18 \div 22°\mathrm{C}$

$P_0 \approx 6$ bar, $D = 10$ mm

Choking occurs when

$p_t/p_0 \approx 0.5$

SINGLE-PHASE GAS FLOW, SHORT NOZZLE



File	M_G kg/h	P_{back} bar
60A10A00.PRE	94.8	0.891
60A13B00.PRE	94.9	1.281
60A19B00.PRE	94.9	1.929
60A33B00.PRE	94.9	3.288
60A41B00.PRE	95.0	4.058
60A47B00.PRE	94.9	4.695
60A56B00.PRE	88.4	5.619

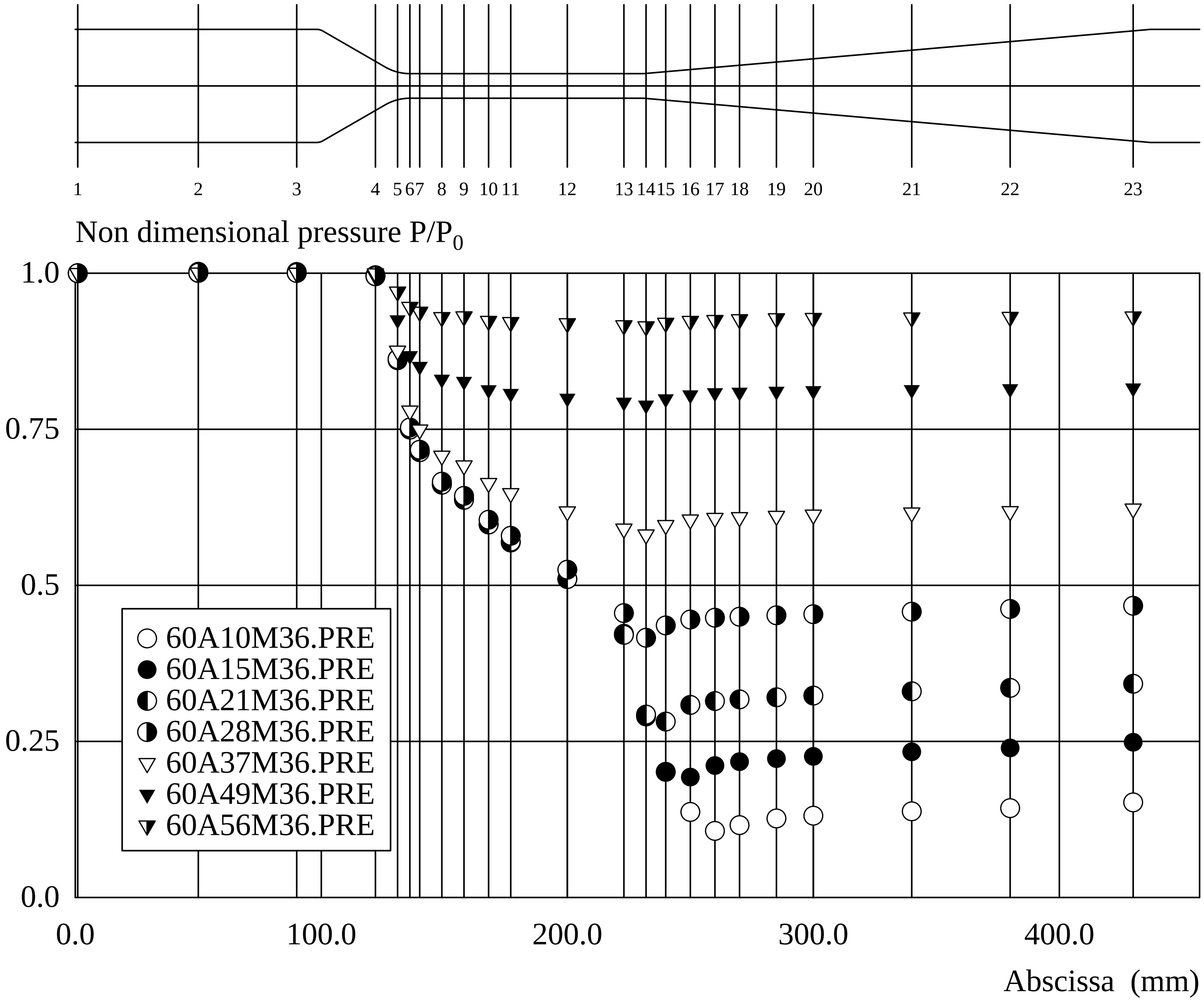
air, $T_G \approx 19°\text{C}$

$P_0 \approx 6$ bar, $D = 5$ mm

Choking occurs when

$p_t/p_0 \approx 0.5$

TWO-PHASE AIR-WATER FLOW



File	M_G kg/h	P_{back} bar
60A10M36.PRE	215.2	0.912
60A15M36.PRE	217.4	1.489
60A21M36.PRE	216.9	2.050
60A28M36.PRE	216.1	2.798
60A37M36.PRE	204.1	3.731
60A49M36.PRE	155.3	4.897
60A56M36.PRE	94.3	5.593

$T_L \approx T_G \approx 19°\text{C}$

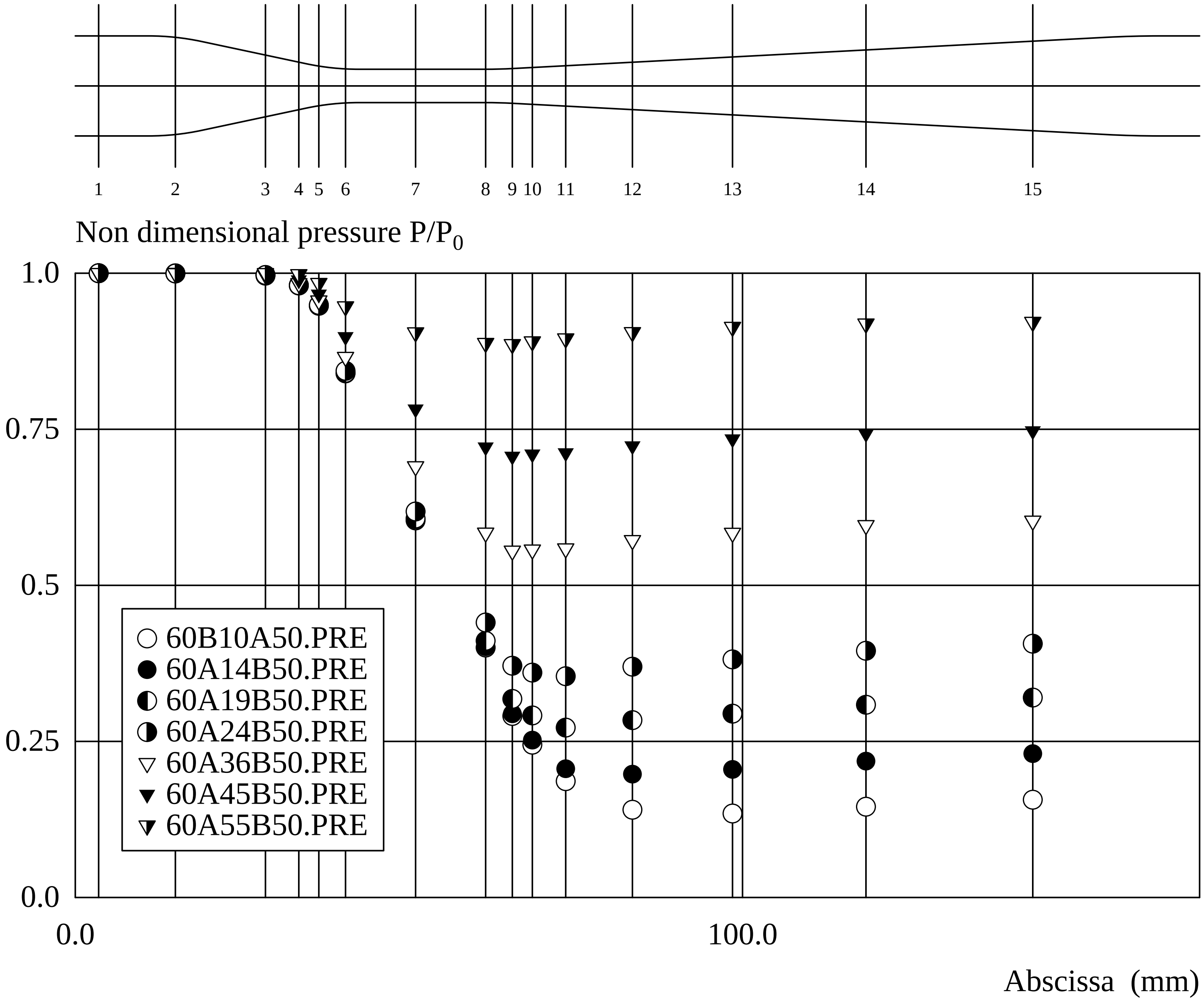
$P_0 \approx 6$ bar, $D = 10$ mm,

$M_L \approx 358$ kg/h.

Choking occurs when

$p_t/p_0 < 0.5$

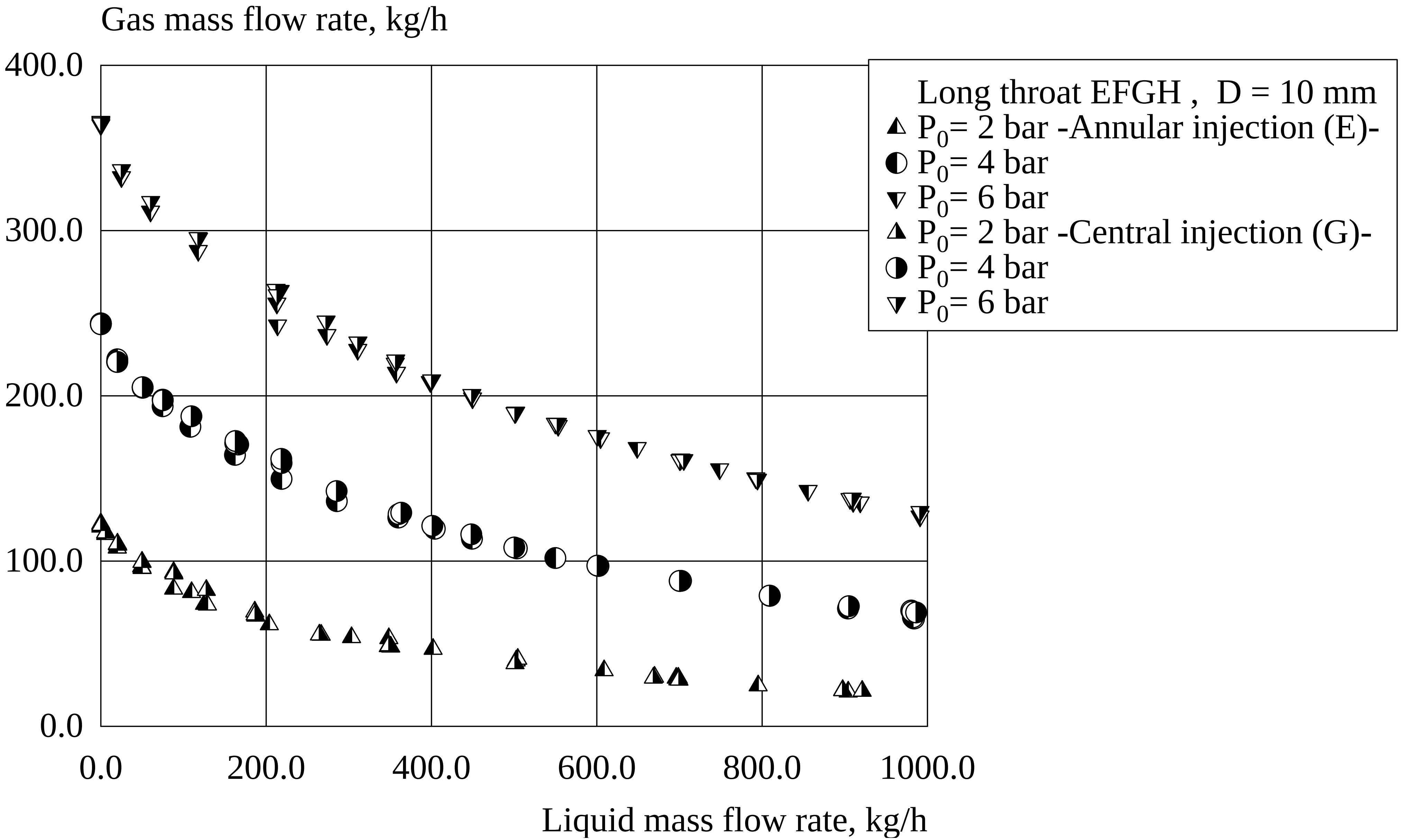
TWO-PHASE AIR-WATER FLOWS



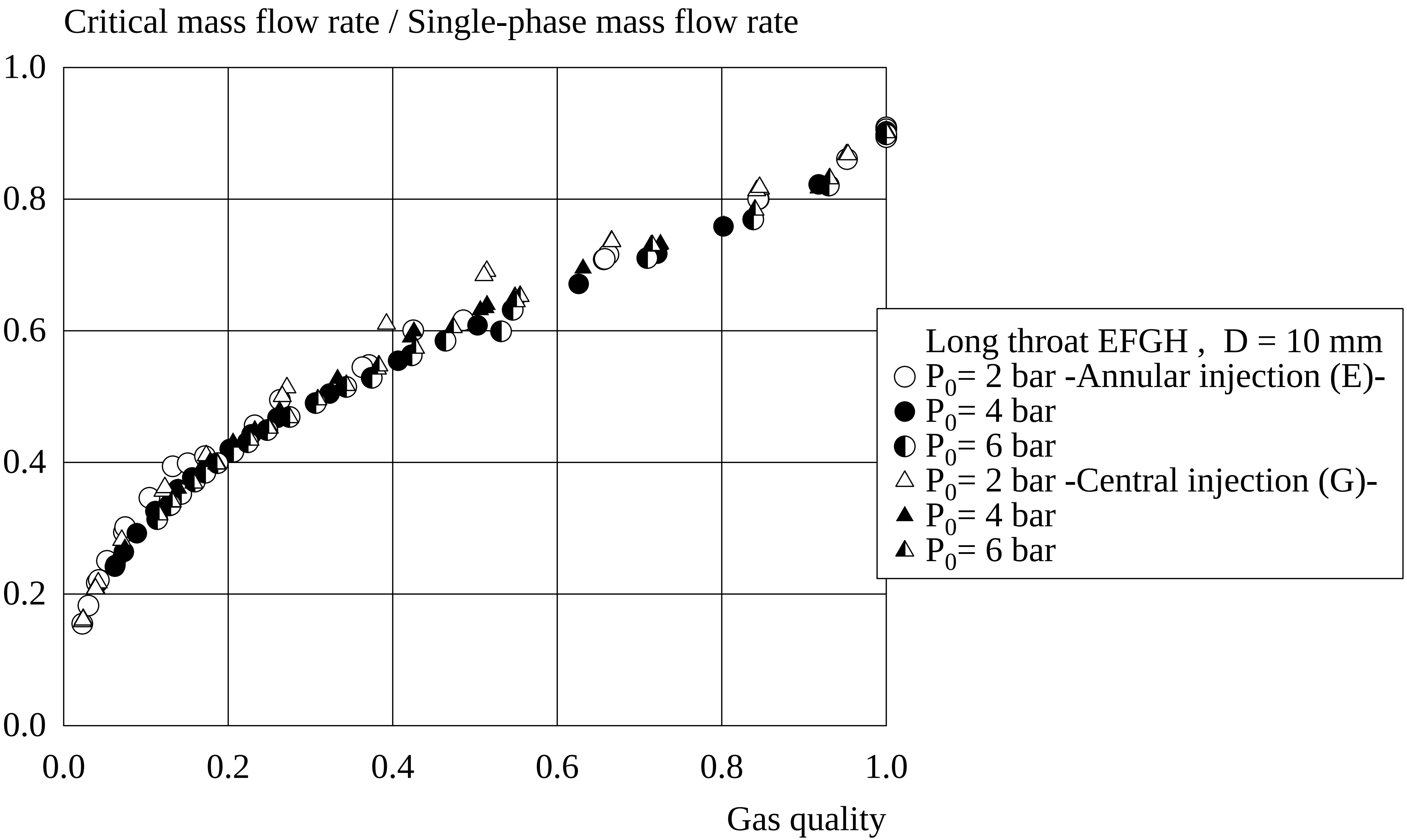
File	M_G kg/h	P_{back} bar
60B10A50.PRE	18.50	0.942
60A14B50.PRE	18.50	1.385
60A19B50.PRE	19.10	1.925
60A24B50.PRE	18.20	2.444
60A36B50.PRE	15.40	3.626
60A45B50.PRE	10.00	4.490
60A55B50.PRE	3.20	5.540

$T_L \approx T_G \approx 19°\mathrm{C}$
$P_0 \approx 6$ bar, $D = 5$ mm,
$M_L \approx 500$ kg/h.
Choking simple criterion lost.

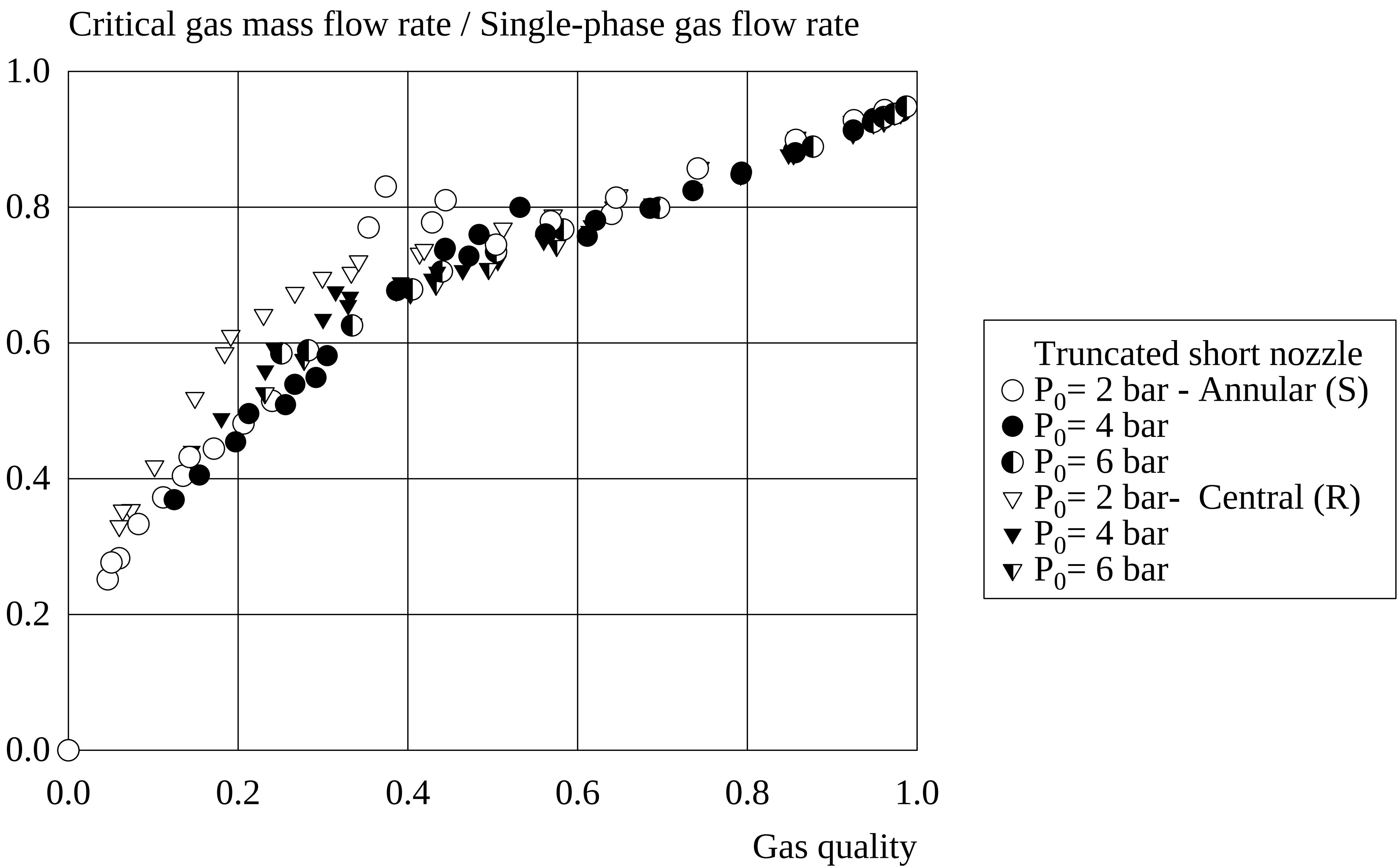
SAFETY VALVE CAPACITY REDUCTION



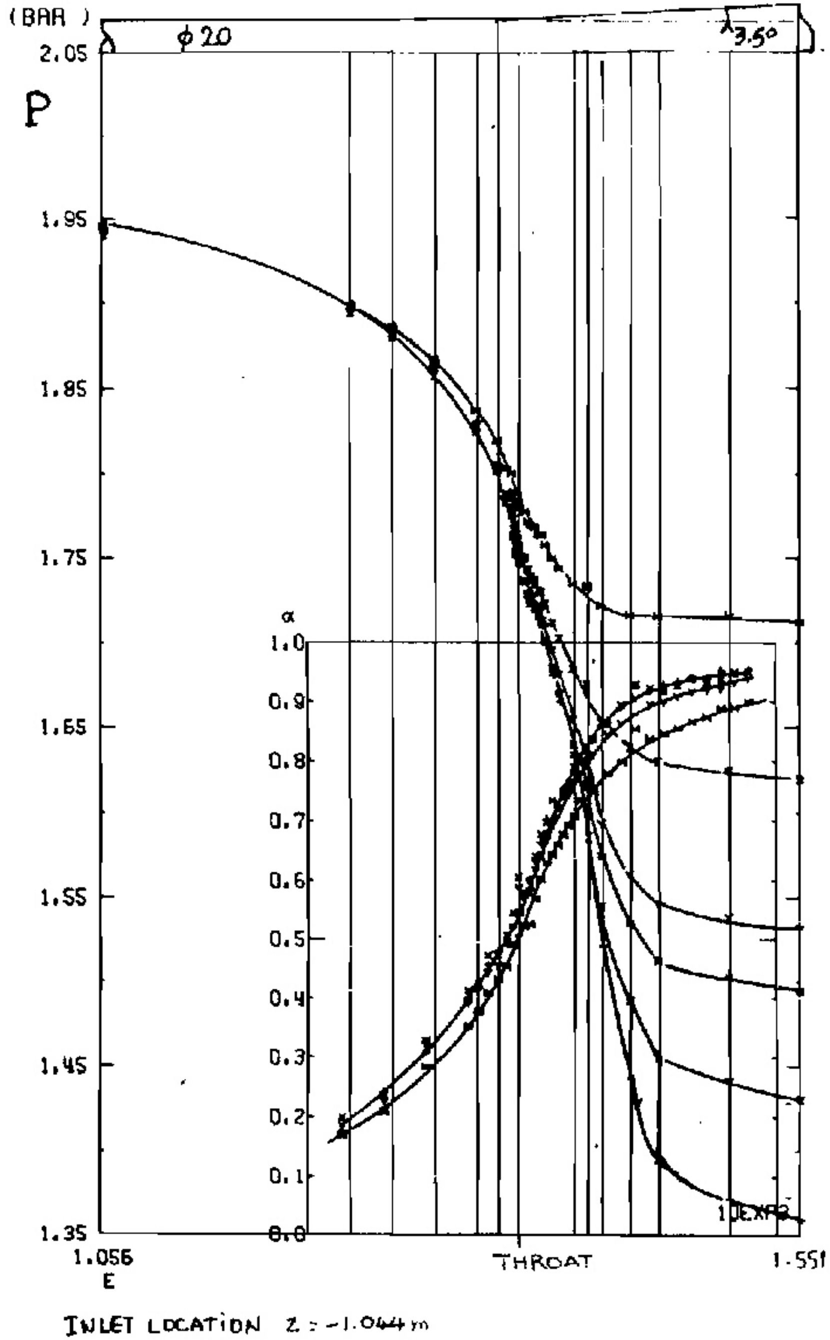
QUALITY EFFECT ON GAS CAPACITY



SAFETY VALVE CAPACITY REDUCTION, SHORT NOZZLE



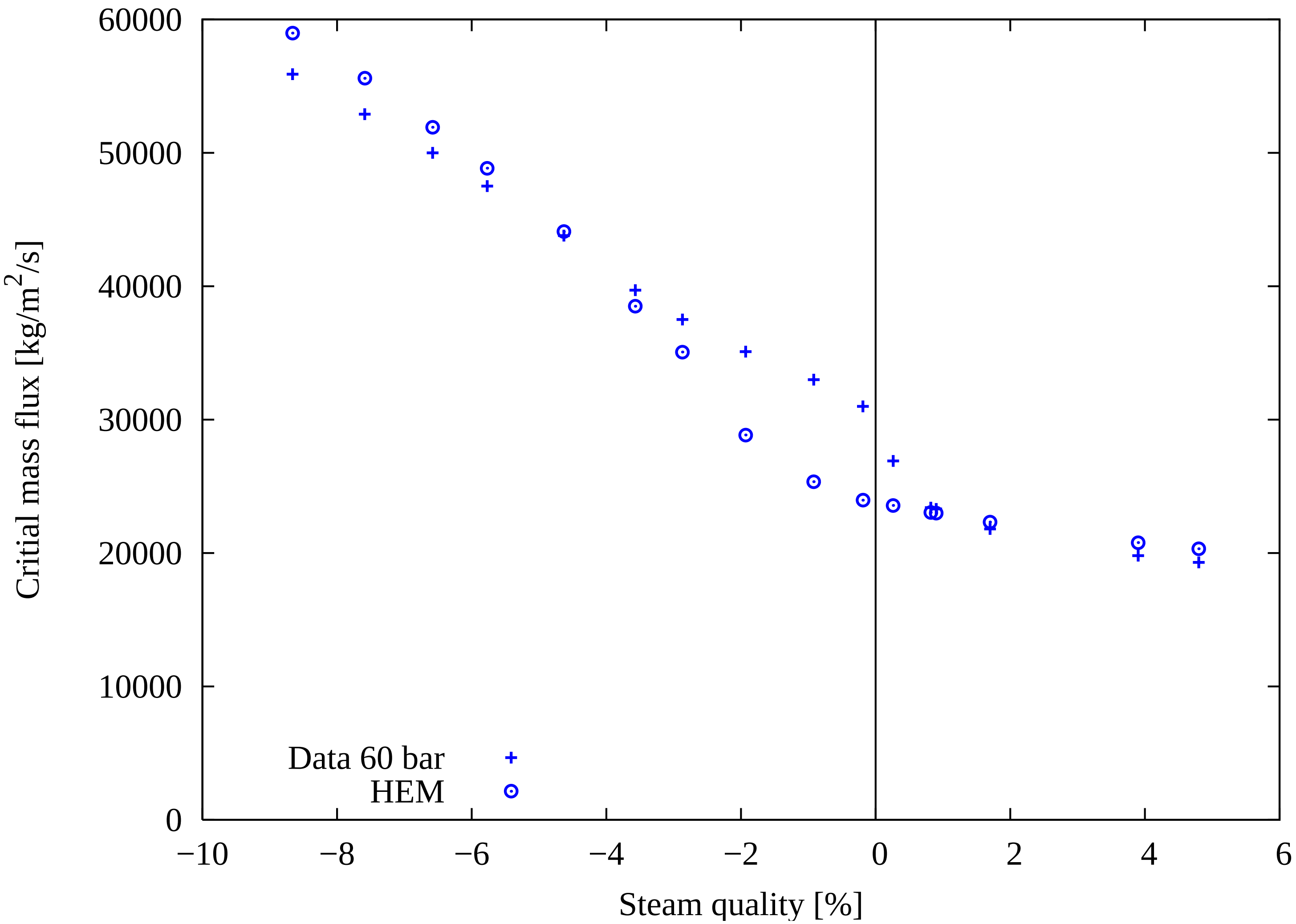
STEAM-WATER FLOWS



Run	Inlet Pressure (bar)	Inlet Temperature (°C)	Downstream Pressure (bar)	Mass flux (kg/m² s)
423	2.294	121.9	1.359	4383
424	2.287	121.8	1.430	4357
425	2.284	121.7	1.494	4355
426	2.285	121.8	1.531	4360
427	2.279	121.8	1.619	4345
428	2.284	121.8	1.712	4331

$$p_{\rm sat}(T_{L0}) = 2.09 \div 2.11 \text{ bar}$$

SUCOOLING EFFET ON CRITICAL FLOW



Super Moby Dick data, 60 bar, saturated and subcooled

In HEM here, friction is neglected.

MAIN FEATURES

- Gas flow rate reaches a limit when the back pressure drops.
- In single-phase flow, this limit depends on
  - Mainly on pressure $M_G \propto SP_0$
  - Geometry, throat length, effect is second order.
- In two-phase flow,
  - The gas flow rate depends on quality.
  - The maximum flow rate of gas and the back pressure for choking depend on geometry,
  - and on inlet effects, mechanical non-equilibrium, $w_G \neq w_L$, history effects.
- In steam water flows, thermodynamic non-equilibrium plays the same role. In flashing flows mechanical non-equilibrium may be secondary.

MODELING OF CHOKED FLOWS

- Single-phase gas or steam and water at thermal equilibrium.
- Theory of choked flows:
  - Time dependent 1D-model, analysis of propagation
  - Stationary flows, critical points of ODE's
- Selected results in two-phase flows,
  - Non equilibrium effects on critical flow.
  - Some numerical results.
- Critical flow is a mathematical property of the 1D flow model.

PRIMARY BALANCE EQUATIONS (1D)

- Mixture mass balance

$$\frac{\partial \rho}{\partial t} + w\frac{\partial \rho}{\partial z} + w\frac{\partial \rho}{\partial z} = -\frac{\rho w}{A}\frac{\mathrm{d}A}{\mathrm{d}z}$$

- Mixture momentum balance

$$\frac{\partial w}{\partial t} + w\frac{\partial w}{\partial z} + \frac{1}{\rho}\frac{\partial p}{\partial z} = -\frac{\mathcal{P}}{A}\tau_W$$

- Mixture total energy balance

$$\frac{\partial}{\partial t}\left(u + \frac{1}{2}w^2\right) + w\frac{\partial}{\partial z}\left(h + \frac{1}{2}w^2\right) = \frac{\mathcal{P}}{A}q_W$$

- Volume forces have been neglected.
- τ_W: wall sher stress, q_W: heat flux to the flow. $\mathcal{P}$: Common wetted and heated perimeter
- Closures must be provided and remain algebraic (no differential terms).

SECONDARY BALANCE EQUATIONS (1D)

- mixture enthalpy balance,

$$\frac{\partial h}{\partial t} - \frac{1}{\rho}\frac{\partial p}{\partial t} + w\frac{\partial h}{\partial z} = \frac{\mathcal{P}}{A\rho}(\tau_W w + q_W)$$

- Mixture entropy balance,

$$\frac{\partial s}{\partial t} + w\frac{\partial s}{\partial z} = \frac{\mathcal{P}}{A\rho T}(\tau_W w + q_W)$$

- NB: secondary equations were derived from primary ones.
- Mixture equations remain valid if mechanical or thermodynamic non-equilibrium are accounted for.

PROPAGATION ANAYSIS

- Mass, momentum, entropy balances for the mixture,

$$\mathbb{A}\frac{\partial \mathbf{X}}{\partial t} + \mathbb{B}\frac{\partial \mathbf{X}}{\partial z} = \mathbf{C}$$

- Equation of state, $p(\rho, s)$, the pressure p should be eliminated.

$$\mathbf{X} = \begin{pmatrix} \rho \\ w \\ s \end{pmatrix}, \quad \mathbb{A} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad \mathbb{B} = \begin{pmatrix} w & \rho & 0 \\ p'_\rho/\rho & 1 & p'_s/\rho \\ 0 & 0 & w \end{pmatrix},$$

- Waves are small perturbations, perturbation method,, $\mathbf{X} = \mathbf{X}_0 + \epsilon \mathbf{X}_1 + \cdots$,
- $\mathbf{X}_0$: Steady state solution.
- Taylor expansion, polynomials in ϵ ...

SOLUTIONS

- Steady flow,

$$\mathbb{B}\frac{\partial \mathbf{X}_0}{\partial z} = \mathbf{C}$$

- Linear perturbation,

$$\mathbb{A}(\mathbf{X}_0)\frac{\partial \mathbf{X_1}}{\partial t} + \mathbb{B}(\mathbf{X}_0)\frac{\partial \mathbf{X_1}}{\partial z} = \mathbb{D}\mathbf{X}_1$$

- RHS are evaluated at $\mathbf{X}_0$,

$$\mathbb{D}\mathbf{X}_1 = \frac{\partial \mathbf{C}}{\partial \mathbf{X}}\mathbf{X}_1 - \frac{\partial \mathbf{A}}{\partial \mathbf{X}}\mathbf{X}_1\frac{\partial \mathbf{X}_0}{\partial t} - \frac{\partial \mathbf{B}}{\partial \mathbf{X}}\mathbf{X}_1\frac{\partial \mathbf{X}_0}{\partial z}.$$

- Perturbation as waves, $\mathbf{X}_1 = \widehat{\mathbf{X}}_1 e^{i(\omega t - kz)}$
- c, phase velocity of small perturbations,

$$c \triangleq \frac{\omega}{k}, \quad \left(c\mathbb{A} - \mathbb{B} - \frac{\mathbb{D}}{ik}\right)\widehat{\mathbf{X}}_1 = 0$$

- Dispersion equation.

DISPERSION EQUATION, SOUND VELOCITY

- Large wave number assumption, $k \to \infty$ ($\mathbf{X}_0$: is quasi uniform),
- Non zero solutions if and only if,

$$\det (c\mathbb{A} - \mathbb{B}) = (w - c)(w^2 - p'_\rho) = 0$$

- 3 propagation modes: w, $w \pm a$, a: so called *isentropic speed of sound*,

$$a^2 = p'_\rho \triangleq \left(\frac{\partial p}{\partial \rho}\right)_s$$

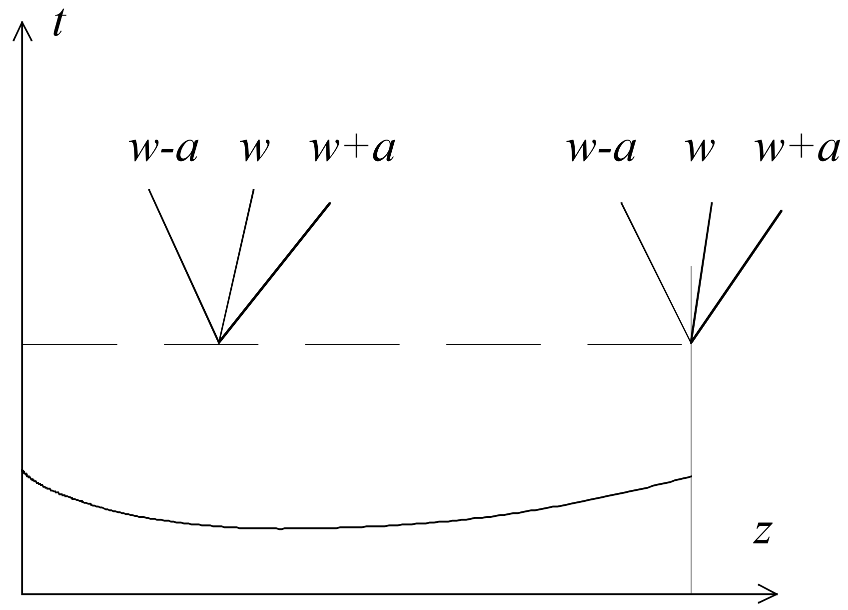
- Examples,
  - Perfect gas, $\widehat{R} = \dfrac{R}{\mathcal{M}}$, R p. g. cst., $\gamma = \dfrac{C_P}{C_V}$,

$$a^2 = \gamma RT$$

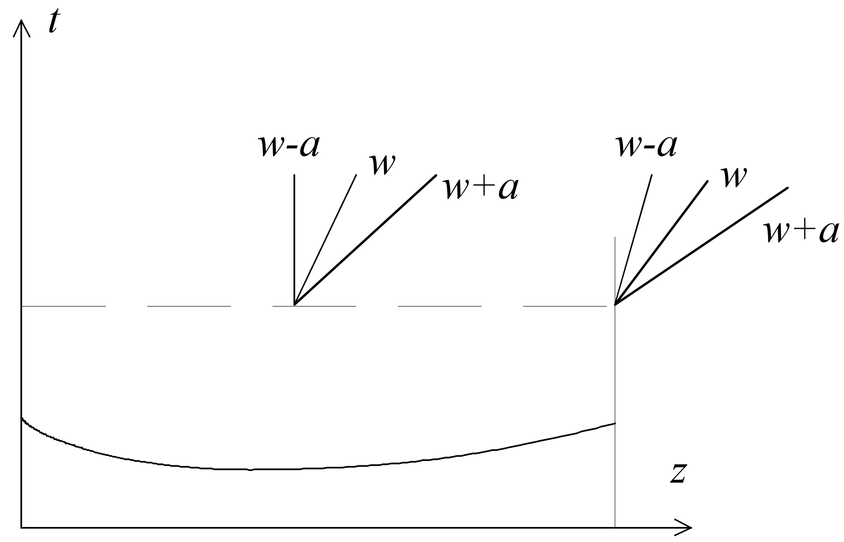
  - Steam and water at thermal equilibrium,

$$a^2 = \frac{h'_x}{\rho'_p h'_x + \rho'_x (1/\rho - h'_p)}, \qquad \begin{cases} h(x,p) = x h_{V\mathrm{sat}} + (1-x) h_{L\mathrm{sat}}, \\ v(x,p) = x v_{V\mathrm{sat}} + (1-x) v_{L\mathrm{sat}} = \frac{1}{\rho} \end{cases}$$

WAVE PROPAGATIONS AND CHOKING



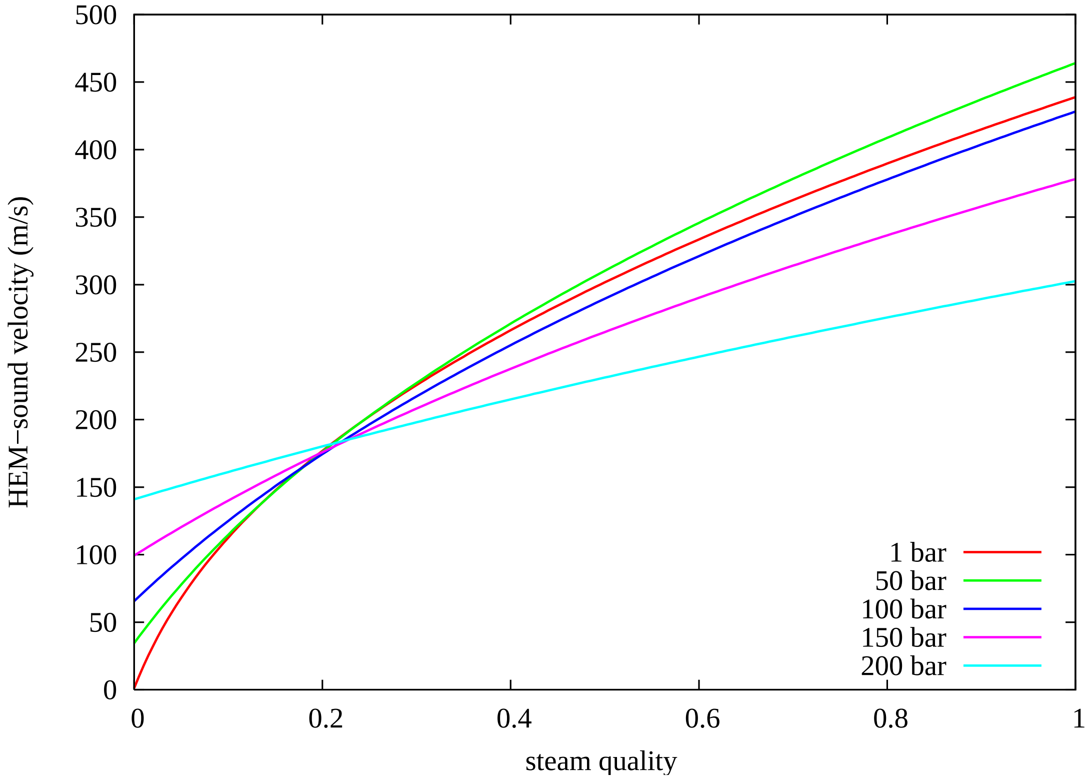
(a) subsonic flow



(b) supersonic flow

- When $w - a < 0$ every where, subsonic flow, flow rate depends on back pressure
- If somewhere, $w - a > 0$, supersonic flow,
  - Waves can no longer propagate from downstream
  - the point where $w = a$ is the critical (sonic) section. Waves are stationary.

THE CRITICAL VELOCITY OF THE HEM



- Air at 20°C,

$$a \approx 343 \text{ m/s}$$

- Steam and water at 5 bar,
  - Thermal equilibrium mixture

$$1 < a < 439 \text{ m/s}$$

  - Saturated Water only at 5 bar $a \approx 1642$ m/s
  - Saturated steam only at 5 bar : $a \approx 494$ m/s
- The mixture compressibility results from the change in composition at thermodynamic equilibrium.

PRACTICAL IMPLEMENTATION

- Transient analysis shortcomings:
  - Physical consistency of the two-fluid one-pressure models
  - Conditionally hyperbolic,
  - Terrible numerical analysis (non-conservative schemes)
  - Time and space requirements are large to resolve waves.
- Critical flow can be analyzed with the stationary model
- Steady equations are much simpler,
  - EDO's instead of EDP's, no physical consistency problems,
  - Initial value problem,
  - Simple and accurate schemes (Runge-Kutta, adaptative step)
  - Price to pay: critical points, where solutions is not unique (*there's no free lunch...*)

STATIONARY FLOW OF A PERFECT GAS

- Mass balance,

$$\frac{\mathrm{d}}{\mathrm{d}z}A\rho w = 0$$

- Momentum balance, circular pipe, C_F is the friction coefficient,

$$\rho w\frac{\mathrm{d}w}{\mathrm{d}z} + \frac{\mathrm{d}p}{\mathrm{d}z} = -\frac{4}{D}\frac{1}{2}C_F\rho w^2 = 0$$

- Energy balance, adiabatic flow,

$$\frac{\mathrm{d}}{\mathrm{d}z}\left(h + \frac{1}{2}w^2\right) = 0$$

- For a perfect gas and a variable section, $D(z)$, Only one ODE,

$$\mathrm{Ma} = \frac{w}{a}, \quad \frac{\mathrm{d}M^2}{\mathrm{d}z} = \frac{4M^2(1 + \frac{\gamma-1}{2}M^2)(\gamma C_F M^2 - D')}{D(1 - M^2)}$$

SOLUTIONS ANALYSIS

- Variable section nozzle, $D = F(x)$, $y = \mathrm{Ma}^2$,

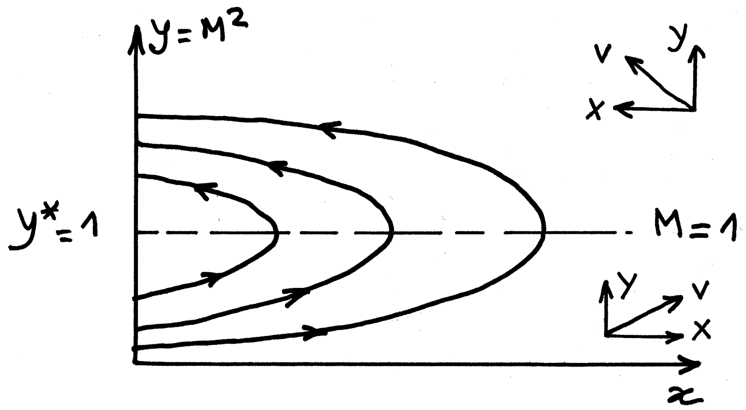
$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{Y(x,y)}{X(x,y)} = \frac{4y(1+\frac{\gamma-1}{2}y)(\gamma C_F y - F')}{F(1-y)},$$

- Autonomous form,

$$\begin{cases} \dfrac{\mathrm{d}x}{\mathrm{d}u} = X(x,y) \\ \\ \dfrac{\mathrm{d}y}{\mathrm{d}u} = Y(x,y) \end{cases}$$

- u dummy, advancement parameter, the system is autonomous when u is not explicit in the RHS. $u > 0$ selected, signe of X.
- Solve the initial value problem: Draw the current lines of vector (X, Y). The analysis of the solution topology does not require to calculate them!

ADIABATIC FLOW WITH CONSTANT SECTION,



- Constant section, F =cst, C_F =cst.

$$X = F(1-y) \text{ and } Y = 4\gamma C_F y^2 \left(1 + \frac{\gamma - 1}{2} y\right).$$

- Solution analysis, signs of **X** and **Y**.
- back to EDO's, integration is possible,
- Evolution equation, Ma $\rightarrow M$

$$\frac{1 - M^2}{M^4 (1 + \frac{\gamma-1}{2} M^2)} \mathrm{d}M^2 = \frac{4\gamma C_F}{D} \mathrm{d}z.$$

- Initial conditions, $z = 0$, $M = M_0$

$$\frac{4 C_F z}{D} = G(M) - G(M_0), \; G(M) = \frac{\gamma + 1}{2\gamma} \ln \frac{1 + \frac{\gamma-1}{2} M^2}{M^2} - \frac{1}{\gamma M^2}.$$

- for given $M = M_0$, z cannot exceed z^*, limiting length.

$$\frac{4 C_F z^*}{D} = G(1) - G(M) = \frac{1 - M^2}{\gamma M^2} + \frac{\gamma + 1}{2\gamma} \ln \frac{(\gamma + 1) M^2}{2(1 + \frac{\gamma-1}{2} M^2)}$$

ADIABATIC FLOW WITH FRICTION: FANNO FLOW

- Because of friction the flow is not isentropic, M evolution parameter,
- Velocity

$$\frac{v}{v^*} = M\sqrt{\frac{\gamma+1}{2(1+\frac{\gamma-1}{2}M^2)}}$$

- Temperature,

$$\frac{T}{T^*} = \frac{\gamma+1}{2(1+\frac{\gamma-1}{2}M^2)}$$

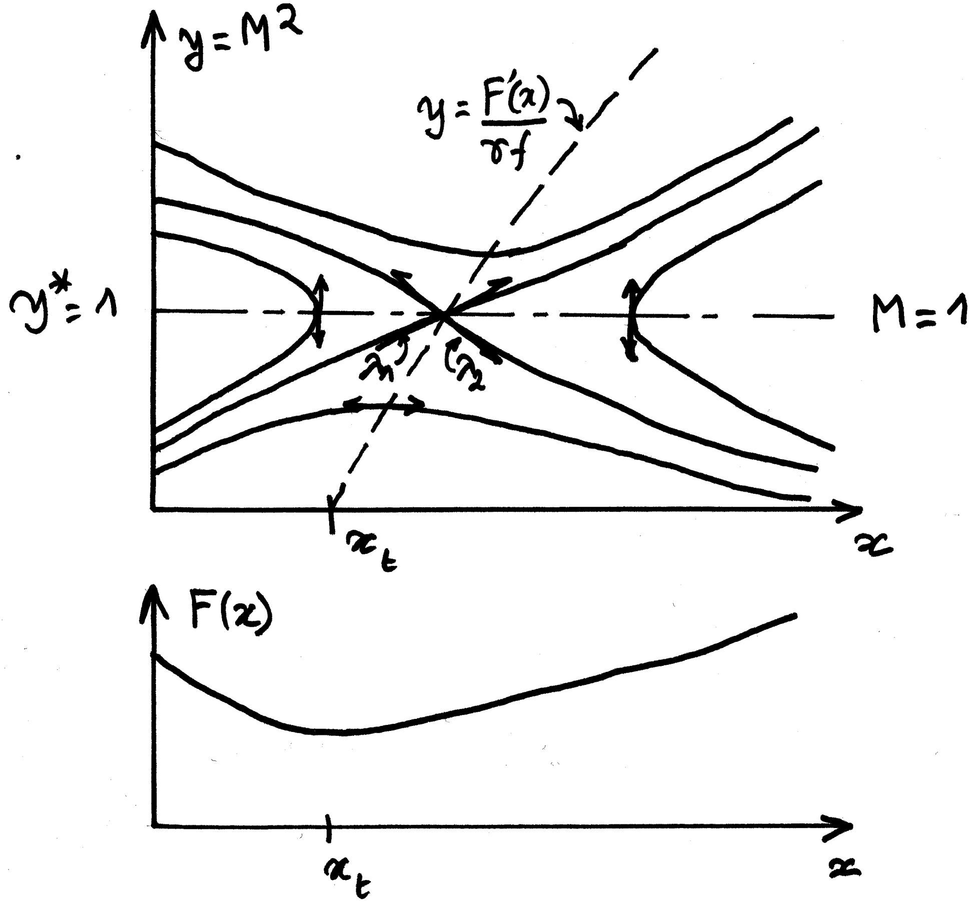
- Density

$$\frac{\rho}{\rho^*} = \frac{1}{M}\sqrt{\frac{2(1+\frac{\gamma-1}{2}M^2)}{\gamma+1}}$$

- Pressure,

$$\frac{p}{p^*} = \frac{1}{M}\sqrt{\frac{\gamma+1}{2(1+\frac{\gamma-1}{2}M^2)}}$$

ADIABATIC FLOW WITH VARIABLE SECTION



- Variable section, adiabatic flow,

$$X = F(1 - y)$$

$$Y = 4y(1 + \frac{\gamma - 1}{2}y)(\gamma C_F y - F')$$

- No analytic solution,
- Signs of **X** and **Y**, 4 quadrants,

$$X = 0 \Rightarrow y = 1, \quad Y = 0 \Rightarrow F'(x) = \gamma C_F$$

- Critical point: **X** and **Y** are both zero. (x^*, y^*) downstream the throat.
- Linear analysis around the critical point.

CRITICAL POINTS FEATURES

- Linearize around critical point, change of variable,

$$\frac{\mathrm{d}y'}{\mathrm{d}x'} = \frac{Y_1}{X_1} = \frac{Y_x(x^*, y^*)x' + Y_y(x^*, y^*)y'}{X_x(x^*, y^*)x' + X_y(x^*, y^*)y'}, \quad \begin{cases} x' = x - x^* \\ y' = y - y^* \end{cases}$$

- Slope of solutions at the critical point, $\lambda = y'/x' = Y_1/X_1$,

$$F(x^*)\lambda^2 + 2C_F\gamma(\gamma+1)\lambda - 2(\gamma+1)F"(x^*) = 0$$

- 2 real roots since F"(x*),

$$\Delta = 4\gamma^2(\gamma+1)^2C_F^2 + 8(\gamma+1)F(x^*)F"(x^*), \quad \lambda_1\lambda_2 = \frac{-2(\gamma+1)F"(x^*)}{F(x^*)}$$

- At the critical point, 2 branches one is subsonic the other is supersonic, other critical points may occur (Kestin & Zaremba, 1953).
- For a variable and decreasing back pressure, subsonic flow, critical flow and supersonic flow. Some range of back pressure can not be reached. In agreement with experiments provided that 1D assumption is correct.

ISENTROPIC FLOW WITH VARIABLE CROSS SECTION

- Very important particular case: isentropic flow.

$$\frac{(y-1)dy}{2y(1+\frac{\gamma-1}{2}y)} = \frac{2F'dx}{F} = \frac{dA}{A}$$

- No history effect ($C_F = 0$), A is the main variable, no longer z. Critical points can only be at the throat or at the end.
- Evolution,

$$\frac{A}{A^*} = \frac{1}{M}\left[\left(\frac{2}{\gamma+1}\right)\left(1+\frac{\gamma-1}{2}M^2\right)\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

- Pressure,

$$\frac{p_0}{p} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}, \quad \frac{p^*}{p_0} = \left(\frac{2}{\gamma+1}\right)^{\frac{\gamma}{\gamma-1}} \approx 0,5283$$

- Mass flux,

$$G = \rho w = p_0\sqrt{\frac{\gamma}{RT_0}}\frac{M}{(1+\frac{\gamma-1}{2}M^2)^{\frac{\gamma+1}{2(\gamma-1)}}}, \quad G^* = \frac{p_0}{\sqrt{RT_0}}\sqrt{\gamma\left(\frac{2}{\gamma+1}\right)^{\frac{\gamma+1}{\gamma-1}}}$$

STEAM WATER FLOW WITH THE HEM

- Important and simple particular case: isentropic flow with saturated reservoir conditions, $p_0, x_0 \rightarrow h_0, s_0$
- Mass, energy and entropy for the mixture, closed form solution,

$$m = S\rho w,$$

$$h_0 = h + \frac{1}{2}w^2,$$

$$s_0 = s.$$

- Mixture variables, thermodynamical variables at saturation

$$\frac{1}{\rho}(x,p) = \frac{x}{\rho_V(p)} + \frac{1-x}{\rho_L(p)} = v(x,p) = xv_V(p) + (1-x)v_L(p)$$

$$h(x,p) = xh_V(p) + (1-x)h_L(p),$$

$$s(x,p) = xs_V(p) + (1-x)s_L(p),$$

A SIMPLE ALGORITHM

- Look for the back pressure, p, that makes $G = \rho w$ maximum,
- Get the quality from entropy,

$$x = \frac{s_0 - s_L(p)}{s_V(p) - s_L(p)}$$

- get the velocity from energy

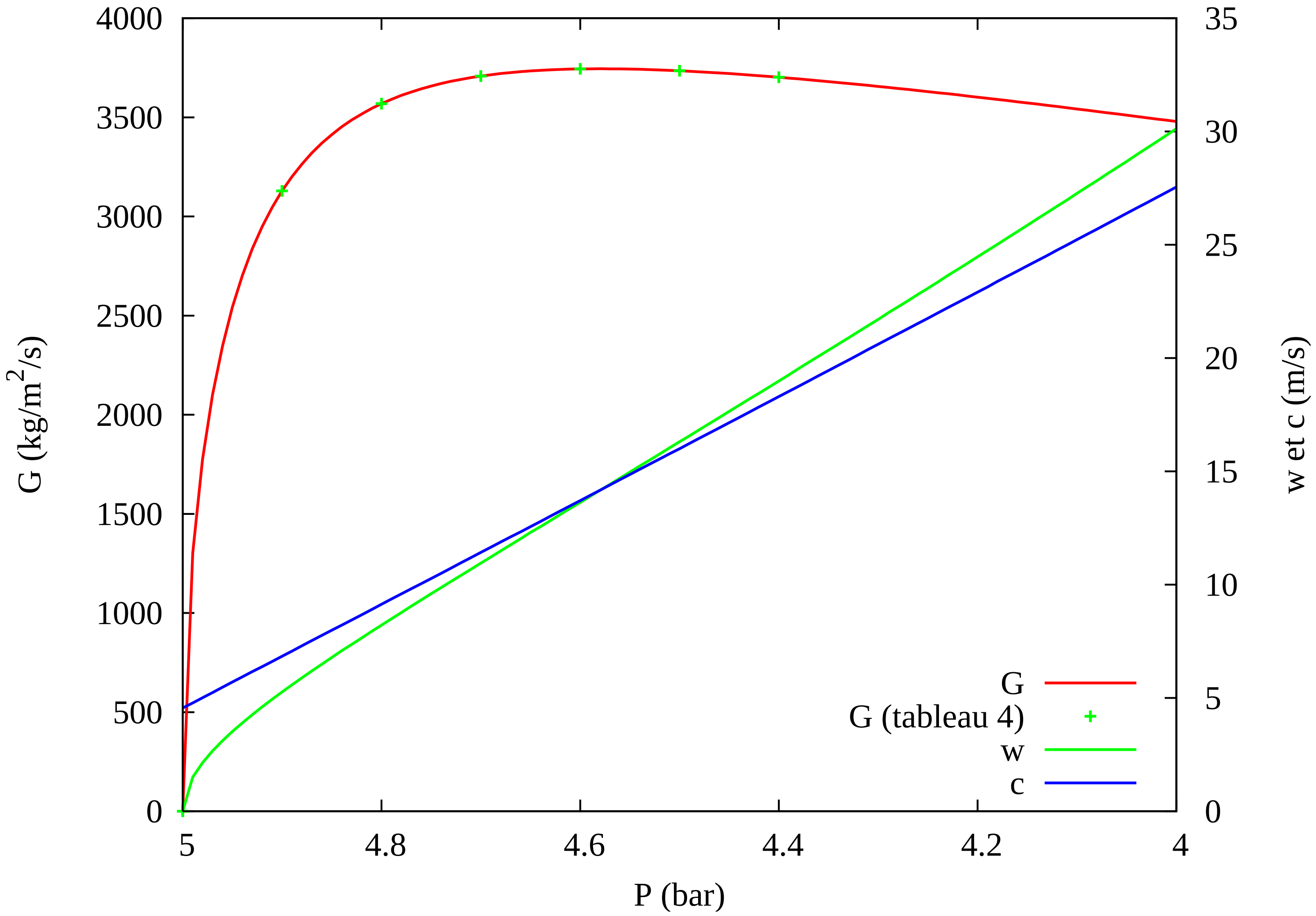
$$w = \sqrt{2(h_0 - h)}$$

- Calculates the mass flux,

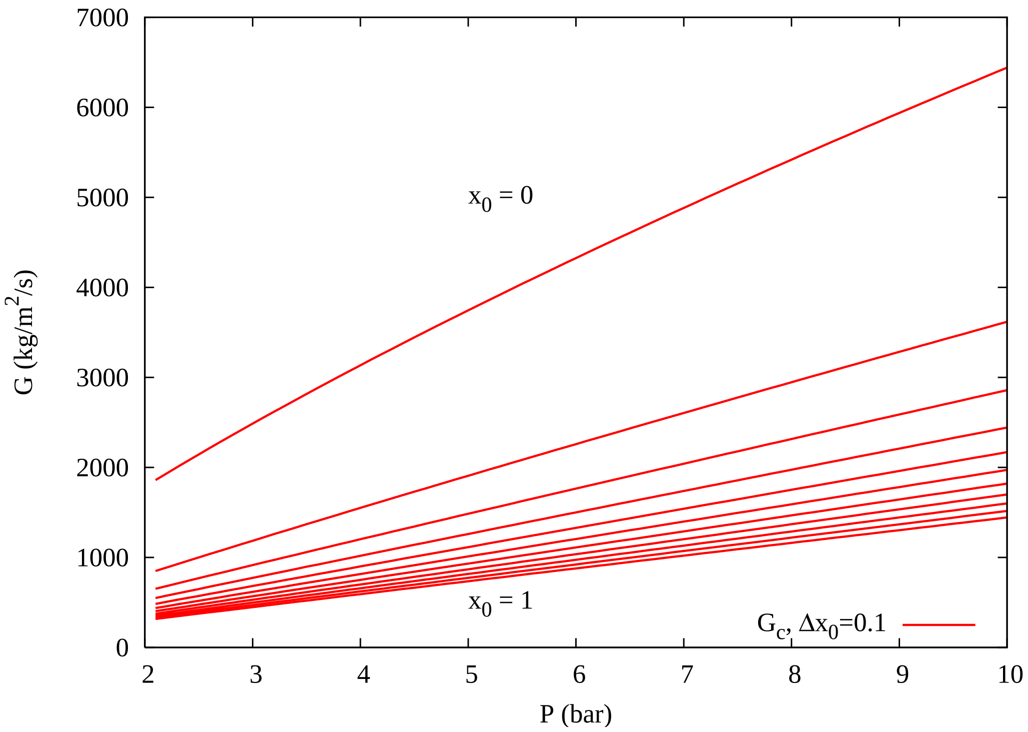
$$G = \rho w = \frac{w}{v}$$

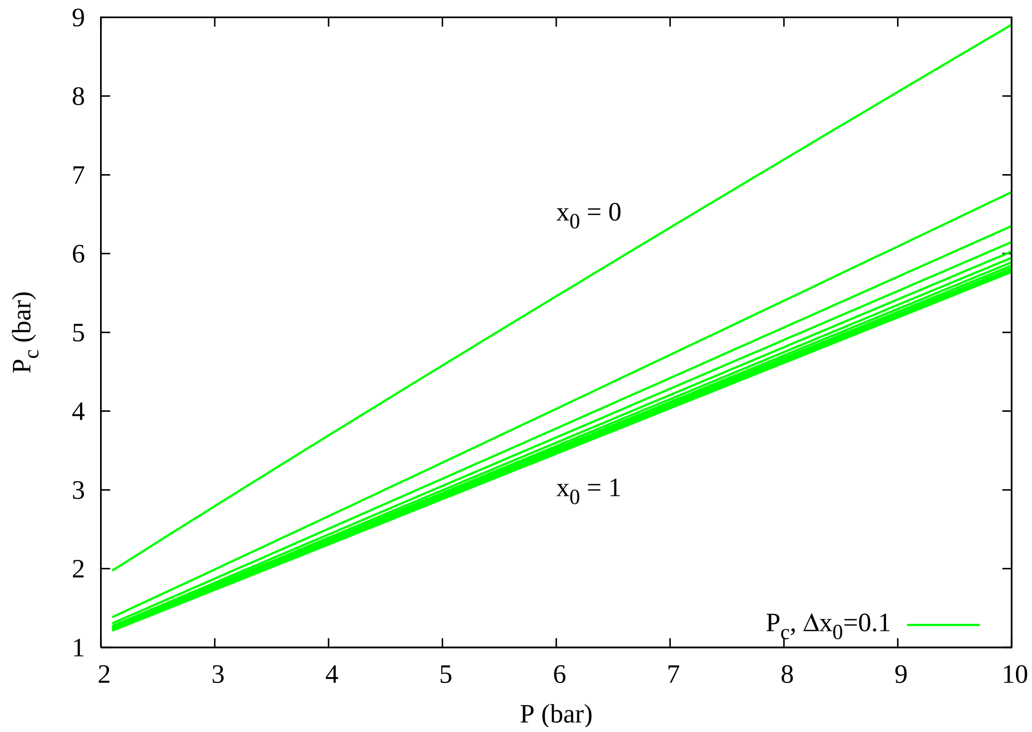
P	x	h	w	ρ	G	c
bar	-	kJ/kg	m/s	kg/m^3	kg/m^2/s	m/s
5.00	.0000	640.38	.00	915.3	.0	4.56
4.90	.0015	640.37	5.26	594.5	3128.7	6.84
4.80	.0031	640.35	8.21	434.4	3568.6	9.13
4.70	.0047	640.32	10.95	338.5	3707.7	11.42
4.60	.0063	640.29	13.63	274.6	3744.0	13.71
4.50	.0079	640.25	16.31	229.0	3734.7	16.00
4.40	.0096	640.20	18.99	194.9	3702.0	18.30

SATURATED WATER 5 BAR

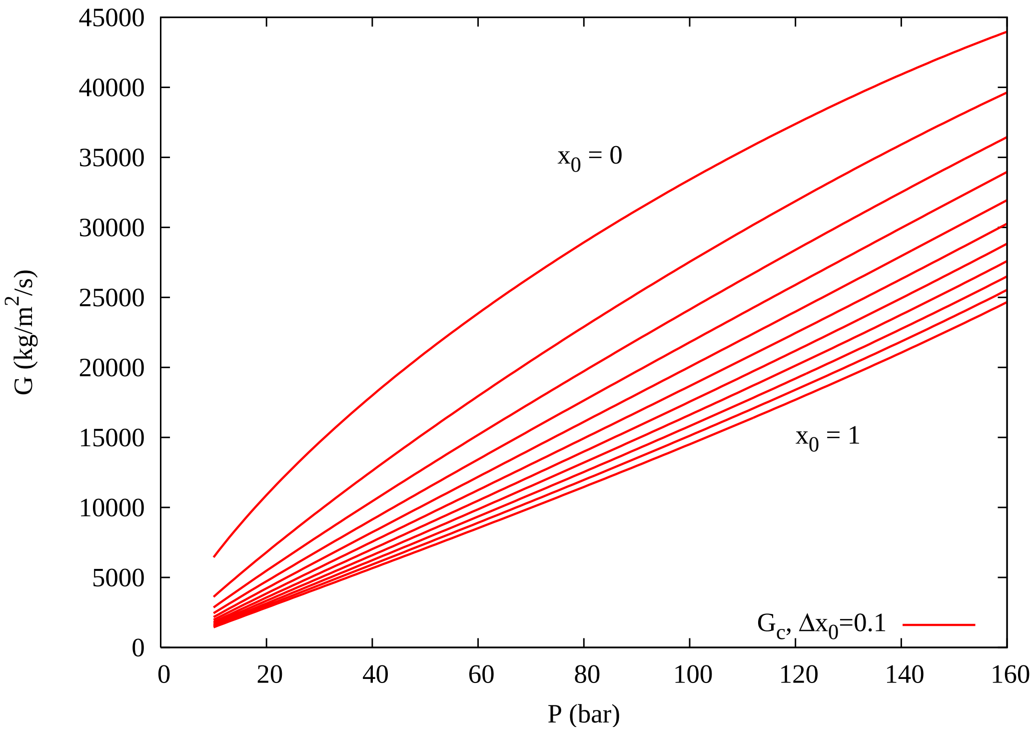


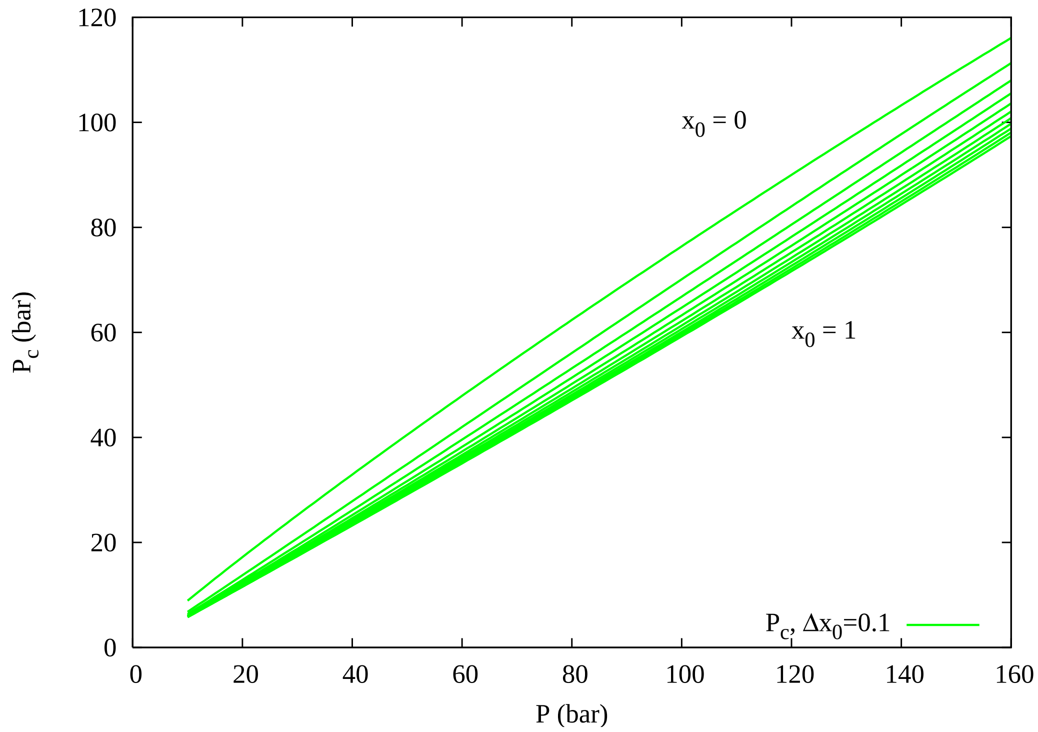
CRITICAL FLOW WITH THE HEM (SATURATED INLET)





CRITICAL FLOW WITH THE HEM (CT'D)





TWO-PHASE FLOW WITH THE TWO-FLUID MODEL

- Two-fluid model, 6 equations or more.
- Wealth of behavior, non-equilibriums, numerical integration is required
- Critical conditions are mathematical properties of the system.
  - The consistency of the velocity propagation depend on the closure consistency.
  - Wave propagation may differ from sound velocity,
  - However the solution topology is identical to that of single-phase flow.
  - The critical section may differ in position (greatly)
- Many choked flow models assume the critical section position, though it results from the calculation
- The critical nature of the flow is assumed, though it derives from the calculation.

CRITICAL POINTS ANALYSIS

- ODE's n equations solved wrt derivatives,

$$\mathbb{B}\frac{\mathrm{d}\mathbf{X}}{\mathrm{d}z} = \mathbf{C}, \quad \frac{\mathrm{d}x_i}{\mathrm{d}z} = \frac{\Delta_i}{\Delta}, \quad i = 1, \cdots n$$

- Autonomous form,

$$\frac{\mathrm{d}z}{\mathrm{d}u} = \Delta, \quad \frac{\mathrm{d}x_i}{\mathrm{d}u} = \Delta_i$$

- Critical point condition,

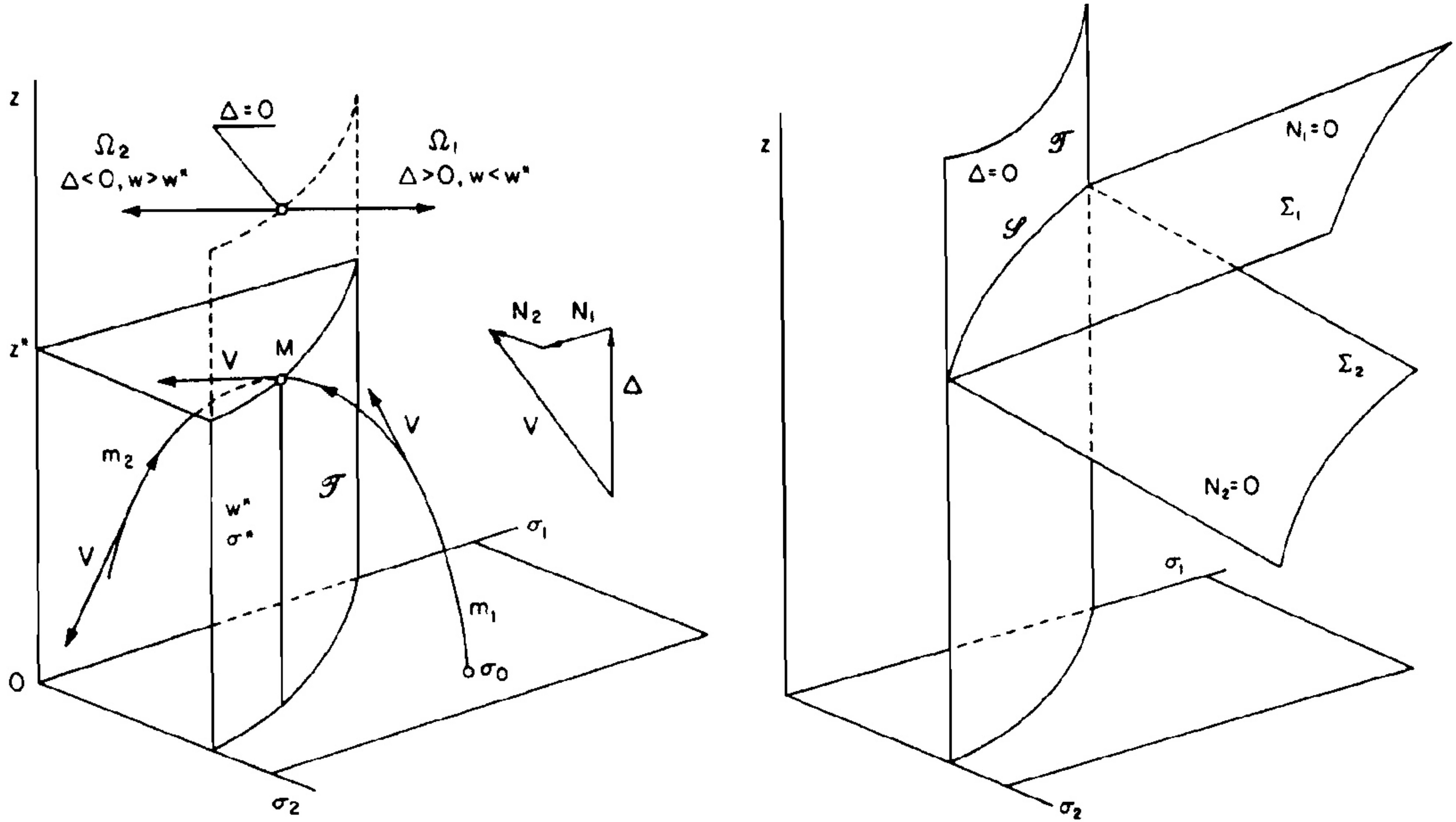
$$\Delta = 0, \quad \Delta_i = 0, \quad i = 1, \cdots n$$

- (Bilicki *et al.* , 1987) showed,

$$\Delta = 0 \text{ and } \Delta_{i_0} = 0 \Rightarrow \Delta_i = 0, \quad i \neq i_0 \in [1, n]$$

- Only two independent critical conditions:
  - $\Delta = 0$, critical condition (i), same as $w = a$ in single phase flow,
  - $\Delta_i = 0$, sets the critical section location, same as $F'(x) = \gamma f$.

SOLUTION TOPOLOGY



After Bilicki *et al.* (1987).

NUMERICAL SOLUTION OF EQUATIONS

- n equations, dimension of phase space is $n+1$
- $\Delta = 0$ ou Δ_i=0 defines a manifold of dimension n.
- All critical points $\in \mathcal{S}$ manifold of dimension $n-1$.
- Topology at the critical point, identical to single-phase flow
- The linearized system has only tow non-zero eigenvalues.
- The corresponding eigenvectors, solution near the critical point.
- One step there and resume numerical integration.
- Shooting problem: Find the point in S connected to $\mathbf{X}_0$ by a solution.

PRACTICAL IMPLEMENTATION

- Boundary problem with a free boundary, $z^* < L$.
- n-1 upstream conditions are given, shooting on the last one (ex. gas flow rate).
- PIF algorithm by Yan Fei (Giot, 1994, 2008)
  - Assume the critical point is a saddle. Dichotomic search.
    - If calculation goes up to the nozzle end, flow is subcritical. increase the gas flow rate.
    - If solution turns back, Δ changes sign $z^* < L$. Decrease the flow rate.
  - This method cannot cross the critical point.
  - Cannot reach the supercritical branch.

- Direct method (Lemonnier & Bilicki, 1994, Lemaire, 1999).
  - Assume there is a saddle. Check later.
  - Find an estimate of critical point by PIF.
  - Set two its coordinates to satisfy exactly. ($\Delta = \Delta_p = 0$).
  - Backward integration (linearization, eigen-values, chek here for the saddle, eigen-vectors...)
  - Solve (Newton) for the remaining $(n-2)$ coordinates to reach $\mathbf{X}_0$
- Allows the full determination of the critical point topology.
- Get the two downstream branches afterwards.
- NB: with non equilibriums, backward integration may become unstable is the system is stiff: change the model...

NON-EQUILIBRIUM EFFECTS

- Thermal non-equilibrium in steam water flows, Homogeneous relaxation model, HRM
  - 3 mixture equations, $x \neq x_{\mathrm{eq}}$, $h_L < h_{L\mathrm{sat}}(p)$, $h_V = h_{V\mathrm{sat}}(p)$

$$\frac{\mathrm{d}x}{\mathrm{d}z} = -\frac{x - x_{\mathrm{eq}}}{w\theta}, \quad \rho = \rho(p, h, x)$$

  - θ is a closure, from Super Moby Dick experiment (Downar-Zapolski *et al.* , 1996).

$$\frac{A'}{A} = \frac{\mathcal{P} C_F w^2}{2A} \frac{\partial \rho}{\partial p} + \frac{x - x_{\mathrm{eq}}}{\theta \rho} \frac{\partial \rho}{\partial x}$$

  - The critical section shifts downstream due to non-equilibrium.
- Mechanical non-equilibrium air-water flows,
  - Two-component isothermal flow
  - Mechanical on equilibrium: liquid inertia and interfacial friction, τ_i

$$\frac{A'}{A} = \frac{\mathcal{P}}{A} \frac{\tau_W}{\rho_G w_G^2} - \frac{3(1-\alpha)\tau_i}{4R_d \rho_G w_G^2} \left(1 - \frac{\rho_G w_G^2}{\rho_L w_L^2}\right)$$

  - The critical section shifts downstream. May leave the nozzle...

MORE ON CRITICAL FLOW

- Two-phase choked flows,
  - Introduction, Giot (1994)
  - Text, Giot (2008), in French
  - Non-equilibrium effects, Lemonnier & Bilicki (1994)
- Voir aussi,
  - Single-phase flow, very tutorial Kestin & Zaremba (1953)
  - Math aspects and critical points, Bilicki *et al.* (1987)
  - Modeling, HRM, Downar-Zapolski *et al.* (1996)
  - Two-component flows, Lemonnier & Selmer-Olsen (1992)

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